Computational Complexity of Cosmology in String Theory

Michael R. Douglas

[1] Simons Center / Stony Brook University

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Abstract

Based on arXiv:1706.06430, with Frederik Denef, Brian Greene and Claire Zukowski.

String theory

String theory is the first and the best candidate we have for a theory underlying all of fundamental physics:

- It unifies gravity and Yang-Mills theories with matter.
- Thanks to supersymmetry, it does not have the UV divergences of field theoretic quantum gravity in $D > 2$, while still preserving continuum spacetime and Lorentz invariance.
- It realizes maximal symmetries and other exceptional structures: maximal supergravity, $N = 4$ SYM, E_8, ...
- It realizes a surprising network of dualities which unify many ideas in theoretical physics.
- Although it is naturally formulated in 10 and 11 space-time dimensions, it is not hard to find solutions which are a direct product of 4d space-time with a small compact space, and for which the effective 4d physics at low energies is the Standard Model coupled to gravity.

String theory has a large number of solutions for the extra dimensions. Some of these lead to the Standard Model field content, but with a range of values for the cosmological constant and other constants of nature.

This enables the anthropic solution to the cosmological constant problem. Anthropic ideas can help answer other questions about “why is the universe suited for our existence?”

It also makes it very difficult to get definite predictions from the theory. To test the theory we want to make predictions for physics beyond the Standard Model. While there are many negative predictions (possible physics which cannot come out of string theory), to make positive predictions we must argue that some solutions are preferred, or at least find a natural probability measure on the set of solutions.

While the string landscape is complicated, there are various axes along which the extra dimensional manifold M and the corresponding vacua can differ, possibly leading to predictions:

- The radius of M or Kaluza-Klein scale R_{KK} is the distance below which gravity no longer satisfies an inverse square law.
- All known families of metastable compactifications are supersymmetric at high energy, but the breaking scale M_{susy} can vary widely. The number distribution is probably $\sim dM_{susy}/M_{susy}$.
- There is a "topological complexity" axis having to do with numbers of homology cycles, distinct branes, and so on: call this number b. This translates into numbers of gauge groups and matter sectors (most of which can be hidden) in the low energy field theory. This number distribution is probably $\sim C^b$ for some $C \sim 10^2$–10^4.
- Idiosyncratic properties of string theory. For example, F theory and heterotic string theory seem to favor GUTs, while intersecting brane models seem to favor three generations of matter.

Although ultimately we would like to study testable predictions from string theory, even reproducing the existing observations is by no means trivial. The most difficult problem is exhibiting a string vacuum which reproduces the observed nonzero value of the dark energy. It is far easier to fit this as a cosmological constant than otherwise. In simplified models of the landscape, most notably the Bousso-Polchinski model, one can argue statistically that such vacua are very likely to exist. This is not the same as exhibiting one.

In 2006 with Frederik Denef, we argued that this may **never** be done: the problem may be computationally intractable.

- Finding local minima in energy landscapes with specified properties is often intractable.
- We showed that the BP model sits in a family of lattice problems which are NP hard.
- Even computing the cosmological constant in a single vacuum is hard, as hard as computing a ground state energy in QFT.

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In other branches of physics, it is usual for a theory to have many solutions – indeed this will be the case for any equation complicated enough to describe interesting dynamics. This is usually handled by making enough observations on a system to narrow down the particular solution which describes it, and perhaps averaging over unimportant degrees of freedom.

There is also usually an *a priori* measure which tells us how likely the various solutions are. For example, when we study the center of the earth (which is far less accessible than particle physics), we assume that it is made of common elements like iron and nickel, not uncommon ones like vanadium and cobalt. This *a priori* measure has both empirical and theoretical support, including our theory of the origin of the elements in stars.

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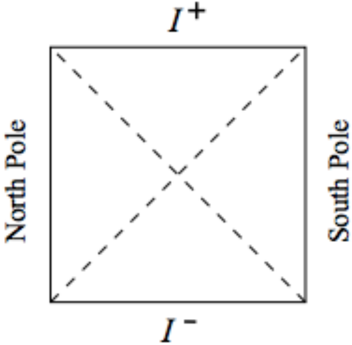
The most basic observations we can make in cosmology are the near-homogeneity and isotropy of the universe, and the deviations from this at order 10^{-5} seen most cleanly in the cosmic microwave background. All of these facts can be explained if we assume a period of inflation in which a positive vacuum energy leads to exponential expansion, roughly modeled by the de Sitter geometry

$$ds^2 = -dt^2 + a^2 d\vec{x}^2 \; ; \qquad a^2 = e^{2Ht} \tag{1}$$

The positive energy must decay at the end of inflation to its small current value and this is most easily obtained by postulating a scalar field ϕ with a potential $V(\phi)$. All of this can easily come out of string theory (and indeed any theory with fundamental scalars). Thus one can try to explain the creation of vacua in string theory by generalizing inflation.

Before explaining this, let us say a few words about de Sitter space-time and quantum gravity. Mathematically, de Sitter and anti-de Sitter are related by exchanging the roles of time and radius. Thus, where AdS has a timelike conformal boundary at large radius *r*, dS has a space-like conformal boundary at large time $\tau \to \pm\infty$, in the infinite past and infinite future. In global coordinates, a constant time surface has the topology of a sphere.



In quantum gravity on a compact space, and in the canonical formalism, it is a bit subtle to define time evolution. Since we will need it shortly, let us explain how this is done.

- The wave function is a functional of spatial metrics and whatever other fields are in the theory, $\Psi[g^{(3)}, \phi]$.
- Since the theory is generally covariant, a change of coordinates is a gauge transformation. In the quantum theory this means that the Hamiltonian and momentum operators act as constraints:

$$\int \delta t(x)\mathcal{H}(x)\Psi = \int \delta\vec{v}(x)\cdot\vec{\mathcal{P}}(x)\Psi = 0. \tag{2}$$

- Thus time evolution must be described by conditioning the wave function on some internal variable which behaves like time. In general one can introduce a "clock." But in cosmology one usually uses the **scale factor** of the metric a^2 as time. In the WKB limit this reduces to evolving a space-like surface defined as $a^2(x) = N^2(t,x)$ for some N.

The theory of inflation, in addition to the slow roll regime for which we have observational evidence, describes a regime of “eternal inflation” in which high c.c. vacua can inflate forever, and in which quantum tunneling produces regions containing all of the vacuum solutions, the “multiverse.”

The multiverse hypothesis can be used to derive a measure. For example, we might postulate that the probability that we live in a specific vacuum is proportional to its space-time volume in the multiverse (the “principle of mediocrity”).

This idea has been studied since the 80’s and there are many results. One of the most important is that – if we choose a time coordinate on the multiverse – we can write an evolution equation for the number (or volume, or weighted volume) of universes at each time. It is linear and the time derivative at t only depends on the number of universes at time t, so it is a Markov process:

$$\frac{d}{dt}N_i = \alpha_i N_i + \sum_j M_{i\leftarrow j}N_j - M_{j\leftarrow i}N_i, \tag{3}$$

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Usually to derive a measure factor one assumes that this Markov process runs to equilibrium, so that N_i will become independent of the initial conditions. As the space-time volume inflates to infinity, this leads to many subtleties. In addition, some choices of the time coordinate lead to paradoxes or contradictions with observation.

One of these is the "youngness paradox" which arises if we take t to be proper time along comoving geodesics. If we exit inflation at $t+\Delta t$ instead of t we find $\exp H\Delta t$ more universes. This favors shortening the history after exit, so observers are predicted to appear as early as possible after the "big bang." Even worse, precise derivations can require postulating a cutoff at the "end of time," so the exponential growth favors appearing as close as possible to the cutoff.

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To remove the youngness paradox one can take “scale factor time” with this inflationary factor removed. Working out the transition matrix and taking the dominant eigenvector which controls the long time limit, one finds a fairly clear prediction (Garriga *et al* 2005, Schwarz-Perlov and Vilenkin 2006):

The measure factor is overwhelmingly dominated by the longest lived metastable de Sitter vacuum. For other vacua, it is given by the tunnelling rate from this “master” vacuum, which to a good approximation is that of the single fastest chain of tunneling events.

Predictions of the equilibrium measure

What is the longest lived metastable de Sitter vacuum? Already on entropic grounds one would expect it to be complicated.

String theory leads to a more specific argument that confirms the expectation that this vacuum will have large b.

The tunneling rate between vacua is $\sim \exp -S_{CdL}$ where S_{CdL} is the action of the Coleman-deLuccia instanton. This depends on the energies V of the initial and final vacuum and the bubble wall tension T roughly as $T^4/(\Delta V)^3$. There are corrections whose most important effects are to suppress tunnelings to higher energy vacua, and make almost-supersymmetric vacua very long lived.

So, the longest lived vacuum has very small $M_{susy} \sim \exp -N/g^2$ if it comes from dynamical breaking with a very small gauge coupling g. Unlike Λ, both M_{susy} and g do not get cancellations, so small M_{susy} comes from an underlying large number of cycles b.

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Thus, the equilibrium measure predicts that we are likely to live in a vacuum which is easy to reach from one of large *b*. Tunneling events are local in the extra dimensions and thus each one makes a small change to *b*. Combining this with entropic considerations, our vacuum should have large *b*.

This may eventually lead to testable predictions. For example, it has been argued (Arvanitaki *et al*, 2010) that string theory can naturally lead to an "axiverse" with hundreds of axions, each associated with an independent homology cycle. Clearly large *b* should favor this. Is this a likely prediction of string theory?

A philosophical reason to doubt this is that it goes against Occam's razor and the history of science. Suppose we found a concrete vacuum which also led to the Standard Model and solved the c.c. problem, say using intersecting branes on a torus. While one might think this would be a good candidate for our universe, since it has small *b*, the equilibrium measure disfavors it. Should we accept this?

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Computational measure

Can we state axioms which favor **simple** vacua as candidates for our universe?
We need an objective definition of simple.

Here are three ideas:

- The mathematics of extra dimensions suggests measures of simplicity: number of homology cycles, number of branes, other topological invariants. Suggestive but not precise.
- String theory probably has preferred initial conditions, possibly the extra dimensional configurations which lead to the smallest Hilbert spaces. Many analogies, starting with fuzzy S^2 (representations of $SU(2)$). Also not precise yet, but we will grant it.

Whatever the initial conditions may be, they are probably too simple to be an anthropically allowed vacuum - why should they contain the SM? We need dynamics to go from the initial vacua to the candidate vacua.

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Whatever the initial conditions may be, they are probably too simple to be an anthropically allowed vacuum - why should they contain the SM? We need dynamics to go from the initial vacua to the candidate vacua.

- Finally, we can say what it means for the dynamics to be simple. One might relate this to time – a simple vacuum is one which is created early-on in the unfolding of the multiverse. However, there is no preferred time coordinate in the multiverse.

This is where we introduce a new idea. We will define complexity of the dynamics as the complexity of a **simulation** of the dynamics by a hypothetical quantum supercomputer. Thus, we postulate that our universe is one which is easy for such a supercomputer to find. Rather than enumerate the most common anthropically allowed vacua in an aged multiverse, we search for simple anthropic vacua in a youthful multiverse.

To turn this idea into physics, we need to make the idea of simulation precise. We need to say what it means to “find” a vacuum. And, if there are many ways to define these terms, we need to look for whatever common predictions these ideas lead to (if any).

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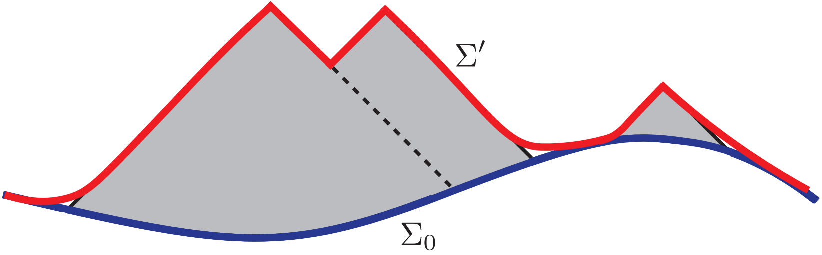
Let us explain how we do this in the semiclassical limit. The state of the multiverse is defined by a 3-metric and fields parameterizing the vacua on a spatial 3-surface Σ_0. Time evolution is generated by a Hamiltonian density $\mathcal{H}$ integrated against a lapse function δt. In the classical limit we could think of this as advancing the spatial surface to Σ' obtained by advancing along geodesics by the local proper time δt.



The primary operation which our quantum supercomputer can perform is to simulate this time evolution. We also allow it to observe the results of its simulation – but it cannot do anything with these observations to change the laws of physics. All it can do is decide which parts of the multiverse to simulate.

Thus, the computer starts with initial conditions on some Σ_0, in which the vacuum is "simple." It then alternates between two operations:

- Make observations to the past of Σ_t and future of Σ_0. These might be 4d experiments, or we might abstract from this the ability to determine that tunneling events have occurred and measure parameters of the new vacuum.
- Based on these observations, choose a δt to advance Σ, by simulating a new region of space-time. In our semiclassical treatment, this usually satisfies Einstein's equations, but occasionally there will be quantum tunneling events creating bubbles of new vacuum.

Clearly there are many details to fill in here. Do we give the computer any prior knowledge about the landscape? Not having much ourselves, we did not.

What is a measurement? We thought of it as a measurement of the low energy spectrum and parameters carried out by some sort of scattering experiments (perhaps carried out behind event horizons so that there is nothing to observe). Clearly the c.c. is important to measure. The overall volume and lifetime is important. If the computer is supposed to judge the suitability of the universe for "life," this might be done by looking for sources of free energy (stars) and spectroscopy (existence of diverse bound states with complicated EM spectra).

Some parameters require a minimum volume to measure – for example the uncertainty principle requires having a volume $V \sim 1/\Delta\Lambda$ to measure the c.c. to $\Delta\Lambda$.
Otherwise it is important that the measurements can be done in a fixed time limit. This is necessary to avoid the youngness paradox.

If we choose δt to reproduce proper time or scale factor time, the evolution of the volume in each type of vacuum will be governed by the Markov process we cited earlier,

$$\frac{d}{dt} N_i = \alpha_i N_i + \sum_j M_{i \leftarrow j} N_j - M_{j \leftarrow i} N_i, \tag{5}$$

But there are other choices – those which depend on the local observations in each vacuum – which lead to modified processes of the same general form, but with the transition matrix M replaced by some $\tilde{M}$.

We can still get a definite measure factor by taking the long time limit of this process. But the measure factor will be different, and it can depend on initial conditions and details of the modifications leading to $\tilde{M}$.

For example,

- The supercomputer can decide that a particular vacuum is "anthropically allowed" – has small c.c., some sort of structure formation and chemistry – and stop, declaring it to be the result of the search. This amounts to making the vacuum terminal, *i.e.* setting the transition rates out to zero.
- It could decide that a vacuum is not fruitful for continuing the search. The CdL tunneling amplitude is typically a double exponential leading to extremely long lifetimes $10^{10^{100}} \ldots$. An efficient search algorithm would not waste time by simulating extremely long-lived vacua for so long. It would switch to other vacua which, while not themselves anthropically allowed, produce new vacua more efficiently.

  To do this in a Markov process one would postulate a rate $\alpha = -r$ at which vacua are dropped from the rate equation. Or, one could instead postulate a tunneling rate r back to the initial conditions.

These modifications will change the long time limit, for many reasons – for example they violate detailed balance.
While there are many choices, the general nature of the resulting measure is fairly universal:

- It is peaked on anthropically allowed vacua which are near the initial conditions.
- The time to reach this modified equilibrium depends more on parameters of the algorithm such as r, than on lifetimes of vacua.

This is to be compared with the equilibrium measure – recall that

- The resulting measure is peaked on vacua which are easily reached from the longest lived metastable vacuum.
- The time to reach equilibrium is roughly the second longest lifetime, a double exponential.

Thus the computational measure does favor simple vacua, and is far more efficient at finding them.

Let us finish by discussing the "efficiency" question and outlining a version of the question "what is the complexity class of cosmology" ? Just for concreteness, let us postulate that the computer can simulate the entire history of the observable universe since the Big Bang, in a second of our subjective time.

But, before doing so, it has to find a compactification which reproduces our four-dimensional laws, or perhaps any "viable" set of laws (definitely small c.c.). And it has to do this by searching through the possibilities (we will be more precise about this shortly).

How long will it take to find a viable vacuum? Will it take of order a "second," or much less time, or much more time?

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The main thing we need to know to make this precise is the computational cost of simulating a given region of space-time. In our early discussions we used the ansatz that a space-time region of volume V would require $\mathcal{C} = M_{Pl}^4 V$ operations to simulate.

In Brown *et al* 1509.07876 it was conjectured that the complexity to produce a state in semiclassical quantum gravity from a reference state is proportional to the action integrated over the region of space-time causally related to the surface where the state is measured – they call on gauge-gravity duality and consider the state of the boundary theory, so it is measured on the boundary.

Although we do not have gauge-gravity duality for semiclassical quantum cosmology, one can make a similar conjecture for the computational cost of simulating a new region of the multiverse.

The main thing we need to know to make this precise is the computational cost of simulating a given region of space-time. In our early discussions we used the ansatz that a space-time region of volume V would require $\mathcal{C} = M_{Pl}^4 V$ operations to simulate.

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Following Brown *et al*, we conjecture that the computational cost of simulating a region R of space-time in semiclassical cosmology is

$$\mathcal{C} = \frac{S}{\pi\hbar} \tag{6}$$

where S is the action integrated over R. In a de Sitter vacuum we have $S \propto \int \Lambda$ so (in line with holography) this is much less than the volume, though still polynomially related.

Now this definition does not always make sense, for example in the Minkowski vacuum the cost would be zero. But the semiclassical cosmologies we want to consider are largely made up of patches of metastable de Sitter. These have positive action and we will argue that in this context, the definition makes sense. It is not obvious because there are AdS bubbles.

According to this definition, the quantum complexity to simulate the observable universe is $\mathcal{C} \sim 10^{120}$. Thus we are asking whether a viable vacuum with c.c. $\sim 10^{-120}$ can be found in this computational time.

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This definition of cost motivates a new definition of global time, which we call "action time" T_A. We postulate an initial value surface Σ_0. The action time of a point p in the future of Σ_0 is then the integral of the action of the intersection of the past causal domain of dependence of p, with the future of Σ_0.

Consider a 4D dS vacuum with metric (in conformal time)

$$ds^2 = L^2 \frac{-du^2 + dx^2}{u^2}. \tag{7}$$

Let the initial slice be at $u = a$, and consider a point P at $x = 0, u = b > a$. The action of spacetime within this past lightcone is

$$T_A \sim M_P^2 L^2 \log(a/b) \sim M_P^2 L^2 \frac{T_P}{L} \tag{8}$$

where T_P is the elapsed proper time.
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As the dynamics proceeds, there will be a chain of tunneling events and a sequence of dS ancestor vacua leading up to a specified point p. Adding up the succession of action times, one finds

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where T_i is the proper time spent in vacuum V_i. Thus the total action will be the total proper time along the path in (varying) Hubble units, weighted by the (varying) number of accessible holographic bits.

In general there will also be tunnelings to AdS bubbles in which the action time does not make sense. These bubbles will crunch and nobody knows quite what they mean in the landscape. Because dS regions can have AdS bubbles in their past, one needs to check that the action time is continuous and monotonically increasing. We have shown that this is the case in the dS regions.

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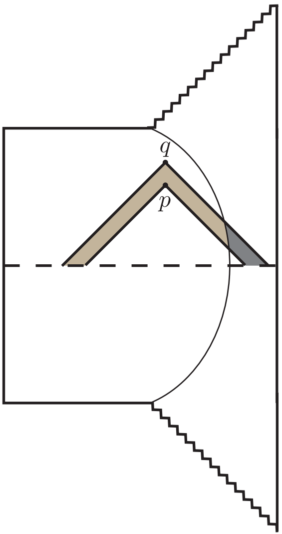

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If the dynamics were deterministic, we could also say that the action time of p is the minimal time needed to **verify** that a proposed cosmology including p satisfies the laws of physics.
This will lead us into the definition of a **complexity class** of a class of vacua in cosmology. Before we talk about this, let us briefly discuss a quantum version of the proposal.

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Quantum gravity

In full quantum gravity, time is a derived concept: the state is a wave functional of 3-geometries (or 9-geometries in string theory) which satisfies the Wheeler-de Witt equation and is thus invariant under time reparameterization.

Given an initial vacuum *init*, we associate a wave function Ψ_{init} which approximately solves the Wheeler-de Witt equation. For each type of vacuum i, there is an operator O_i which is 1 if the state contains the vacuum of type i and 0 otherwise. The natural quantity which measures the probability that the wave function contains vacuum i is then

$$\langle \Psi_{init} | \, O_i \, | \Psi_{init} \rangle \, . \quad (10)$$

Thus this should be the measure factor (after normalization).

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To connect with our previous discussion, we grant that in the semiclassical regime, we have

$$\langle \Psi_{init} | \, O_i \, | \Psi_{init} \rangle = \sum_p \exp -t(p(i)) \, , \tag{11}$$

where t is the action time, in other words it is the sum of terms $\exp -S$ over the causal past of each p which realizes the vacuum i. This sum will normally be dominated by the smallest t and thus the measure will be supported on the vacuum selected by the semiclassical approach.

(There is some sort of analytic continuation being done. It would be interesting to relate the e^{-S} of tunneling amplitudes with the complexity interpretation of e^{iS}.)

All this makes sense in a semiclassical regime which is believed to be the case for inflation. More generally measurements will correlate the quantum states of the multiverse and the supercomputer and one would need to understand this.

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Complexity class of a cosmology

What is the maximal cost $\mathcal{C}_{search}$ (or expected cost) of finding a viable vacuum? We can generalize the problem a bit by looking, not just for $\Lambda \sim 10^{-120} M_{Pl}^4$, but to ask for the cost as a function of Λ. Now we do not know what Λ are attainable in string theory and there are arguments that the list of possibilities is finite (Acharya and Douglas).

But these arguments assume a lower bound on the Kaluza-Klein scale M_{KK} – if we consider decompactification limits we can get arbitrarily small AdS $|\Lambda|$, and plausibly metastable dS as well.

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What is the asymptotic behavior of $\mathcal{C}_{search}(\Lambda)$?

A guess for this is

$$\mathcal{C}_{search}(\Lambda) \sim \frac{1}{\Lambda} \times \mathbb{E}[T(\Lambda)] \tag{12}$$

One factor of $1/\Lambda$ comes if we assume that there is no faster way to find a small c.c. vacuum than by searching at random. The other factor is the average difficulty of actually computing and measuring the c.c. in a given vacuum.

By the uncertainty principle, we need to simulate a space-time volume $1/\Lambda$ to do this at all. The complexity=action hypothesis however allows doing this in $\mathcal{O}(1)$ time. This includes the problem of computing QFT ground state energies, so this claim is probably in tension with complexity of simulating QFT.

Whether we can measure it in this time depends on what we allow as a measurement. In our universe, it is not so obvious that $\Lambda \neq 0$ until it dominates the stress-tensor, as has only been the case recently, cosmologically speaking. This suggests that we need $1/\Lambda$ computation to measure it. We would still have $\mathbb{E}[T] \sim d\Lambda/\Lambda \sim \log \Lambda$.

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This way of phrasing the question singles out Λ so it is not so general. A better way is to say, suppose the cost of simulating a single known universe is $\mathcal{C}_{univ}(\Lambda)$, then what is the relation between $\mathcal{C}_{univ}(\Lambda)$ and $\mathcal{C}_{search}(\Lambda)$? If it is polynomial, then we could say that the problem of finding such a vacuum in string cosmology is in P.

We can also define whether the problem of finding a vacuum from a given set (say, viable) is in NP. It will be if $\mathcal{C}_{search}(\Lambda)$ grows polynomially in $\mathcal{C}_{univ}(\Lambda)$, where we have an **oracle** that always makes the best choices for the search (out of polynomially many). Equivalently, we require that the problem of **verifying** that a cosmology creates the vacuum satisfy the laws of physics be doable in polynomial time.
If we advance the space-like surface Σ_0 everywhere, then if a viable vacuum appears in action time polynomial in $\mathcal{C}_{univ}(\Lambda)$, the problem of finding it would be in NP. This question has the advantage that we don't need to say much about how the search is guided.

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However as stated this would only make sense if the dynamics were deterministic. Of course the dynamics is probabilistic or quantum – so it is better to ask whether the problem of finding a viable vacuum is in BPP or in BQP. These are more or less defined by asking that the probability of finding the vacuum in polynomial time is bounded below by a number greater than $1/2$.

The nondeterministic (or verification) analog of this is the protocol classes MA (Merlin-Arthur) and QMA. Arthur is a computer with a random number generator which can solve polynomial time problems (in BPP) and Merlin is an oracle with infinite computational power. Arthur is allowed to ask Merlin questions about the problem (so, does this candidate cosmology satisfy the laws of physics), and Merlin will answer, but Arthur cannot blindly trust Merlin's answers. If there is a protocol by which Merlin can convince Arthur of the correct answer to a question with high probability, then the problem is in MA.

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Using this definition, we can check whether a class of vacua V_i are in MA by following the time evolution along a sequence of space-like surfaces of increasing action time, and defining a probability distribution over spatial geometries where the probabilities reflect the probabilities of tunneling events between vacua. We define $\mathcal{C}$ to be the time T_A after which probability that a vacuum in the class is created is greater than $2/3$. If $\mathcal{C}$ grows polynomially in $\max_i \mathcal{C}_{univ}(V_i)$, then the class is in MA.

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So, we can ask whether the problem of finding a given class of vacua (say dS with c.c. at most Λ) is in MA or QMA. Even if it is, we can ask whether a particular way to solve the problem attains this theoretical possibility. There are many problems for which a naive algorithm is exponential, and it takes some cleverness to find a polynomial-time algorithm – famous examples are linear programming and testing primality.

So, to summarize the questions we formulated,

1. Is it possible to find a viable vacuum in time polynomial in C_{univ} ?
2. Is it possible to verify the cosmology which finds such a vacuum in polynomial time?
3. Does the usual discussion of eternal inflation find a viable vacuum in polynomial time?
4. Can one at least verify such a cosmology in polynomial time?

We are pretty sure the answer to 4, and thus 3, is NO. We believe the answer to 2 is YES. We don't know the answer to 1.

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