Computational Complexity and High Energy Physics

August 1st, 2017

Rigorous free fermion entanglement renormalization from wavelets

arXiv: 1707.06243

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in collaboration with:

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OVERVIEW

- Tensor networks and quantum field theories
- MERA: quantum circuits, renormalization, wavelets
- One-dimensional Dirac fermions
- Fermi surface: Non-relativistic two-dimensional fermions
- Rigorous approximation result
- Outlook and extensions

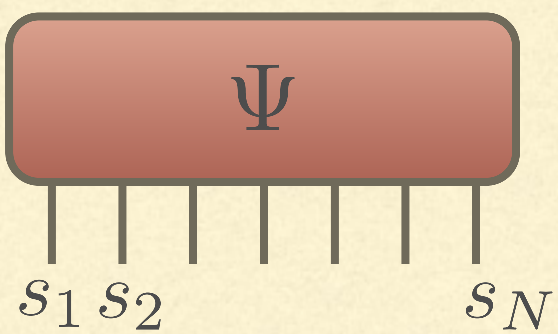
TENSOR NETWORKS ...

Quantum system of *N* spins: $\mathbb{H} = (\mathbb{C}^d)^{\otimes N}$

$$|\Psi\rangle = \sum_{s_n=1}^{d} \boxed{\Psi_{s_1, s_2, \ldots, s_N}} |s_1\rangle \otimes |s_2\rangle \otimes \cdots \otimes |s_N\rangle$$

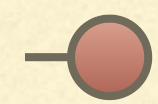
d^N dimensional vector

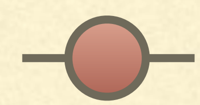
rank *N* tensor

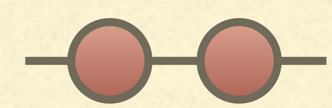


'approximate' with
a tensor network
decomposition

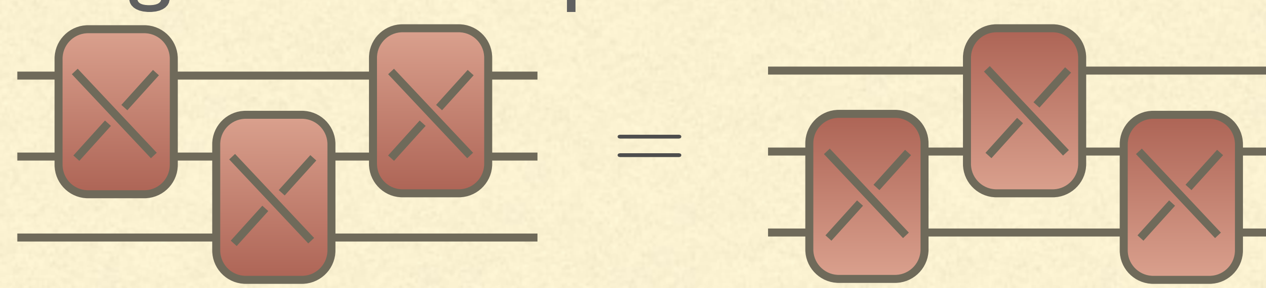
<u>Diagrammatic notation</u>

vector: 

matrix: 

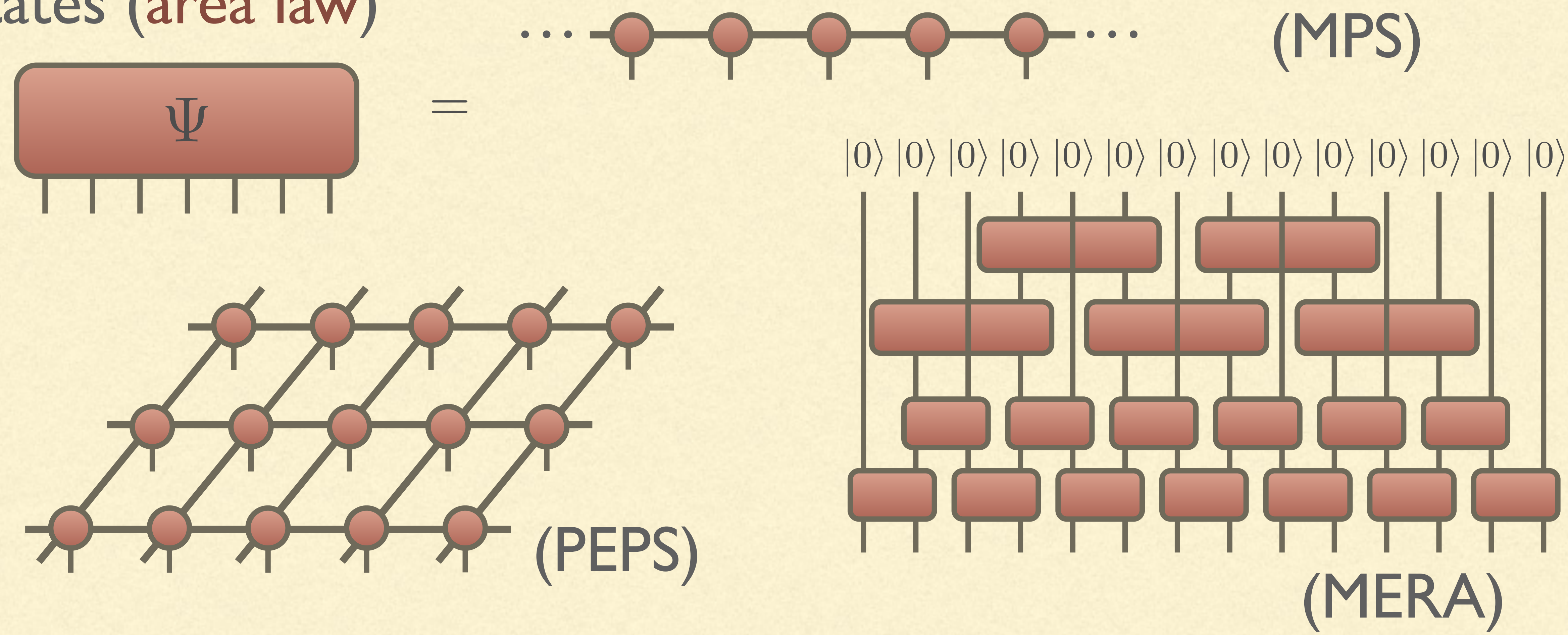
matrix product: 

Yang-Baxter equation:



TENSOR NETWORKS …

- Variational families of states for quantum many body systems, motivated by the structure of entanglement in low energy states (area law)

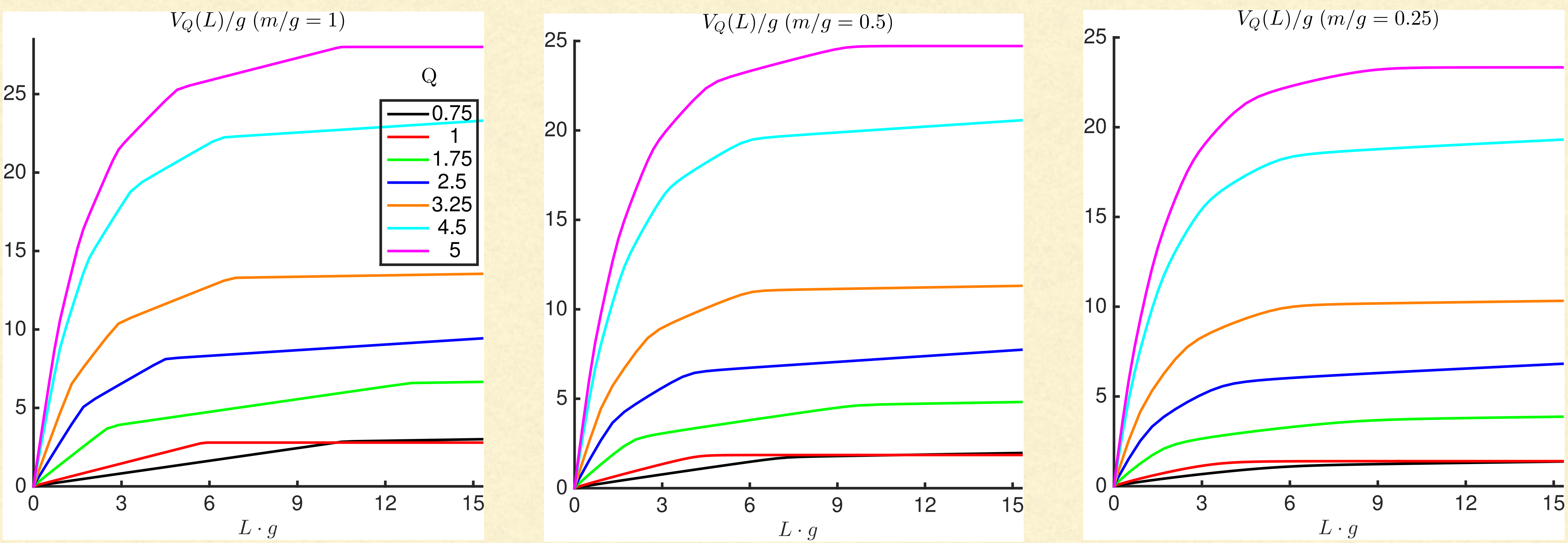


$\Rightarrow$ **classical simulation**

$\Rightarrow$ Complexity scaling (MPS): Hastings; Arad, Kitaev, Landau, Vazirani, Vidick, …

… AND QUANTUM FIELD THEORY (1)

- Variational approach to lattice gauge theory (Hamiltonians)
- Very successful for (1+1)d QFT, e.g. Schwinger model
  TMR Byrnes et al, B Buyens et al, MC Bañuls et al, S Montangero et al, …
  - (Partial) string breaking for heavy probe charges:



Boye Buyens, Jutho Haegeman, Henri Verschelde, Frank Verstraete, Karel Van Acoleyen, PRX 6, 041040 (2016)

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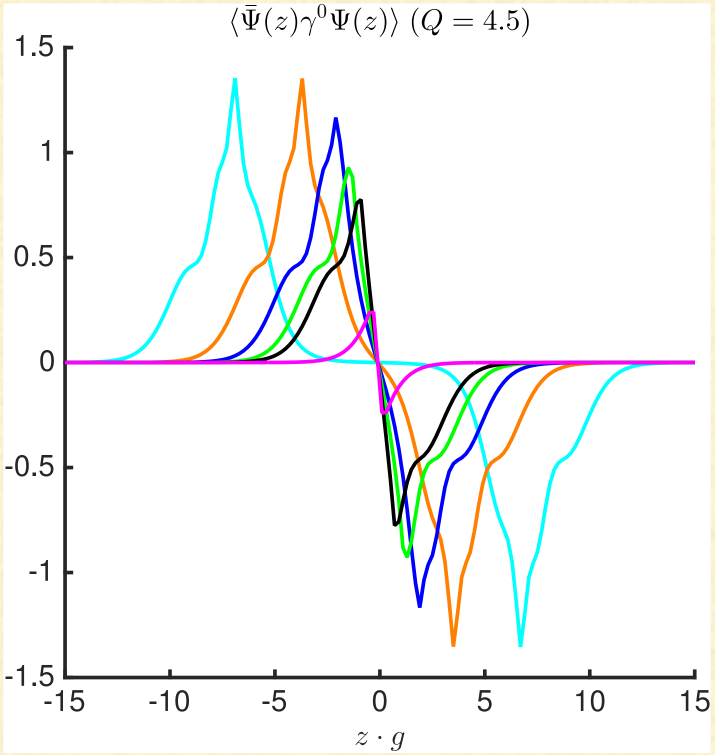

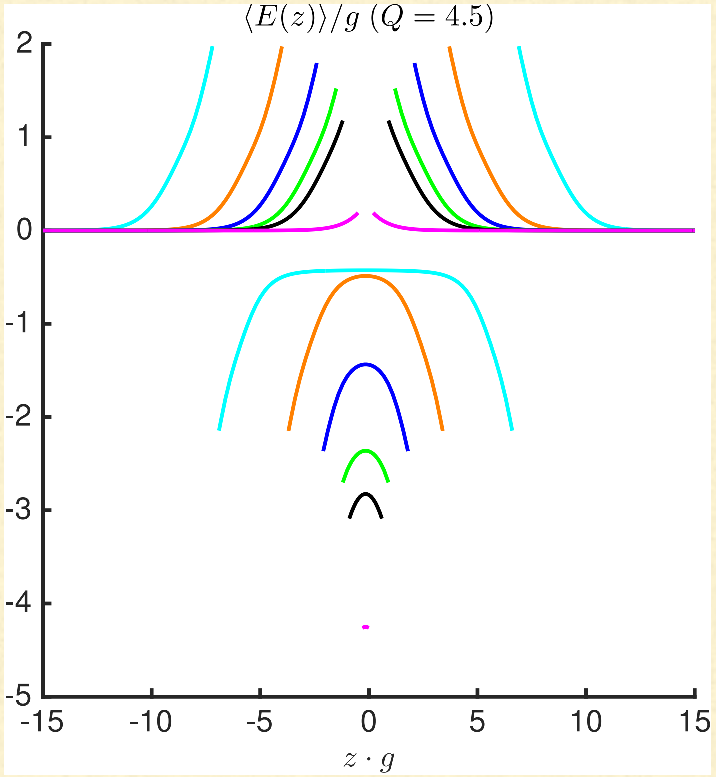

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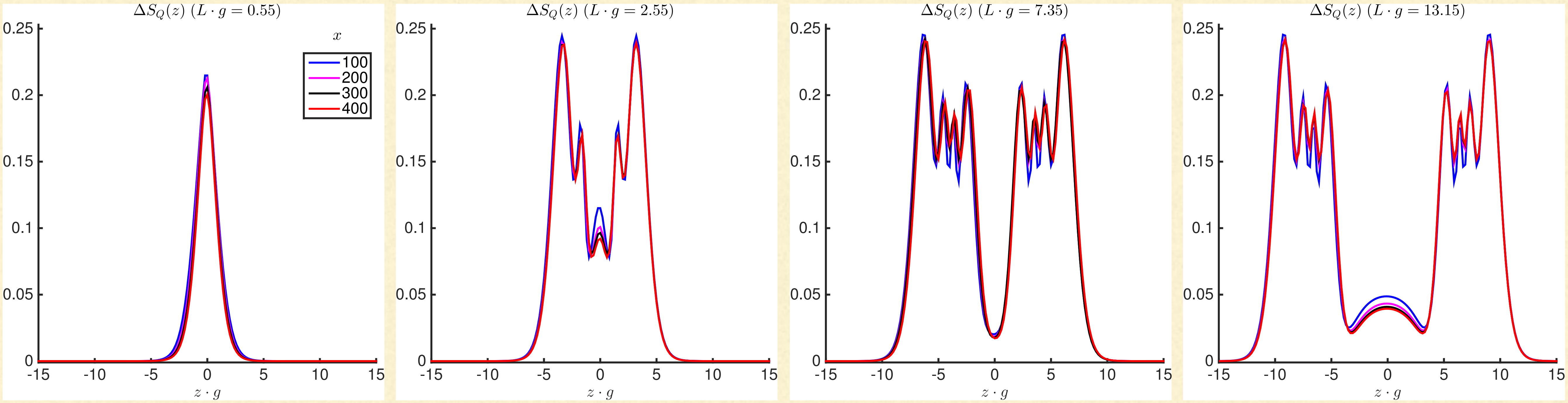

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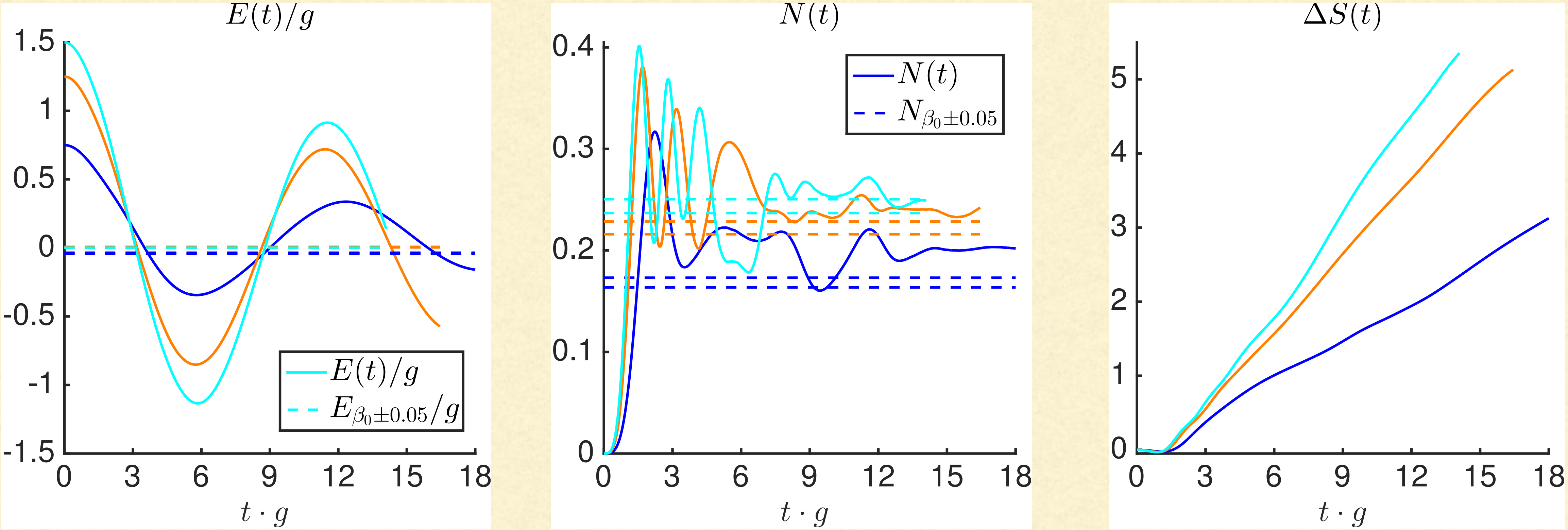
  - Real time evolution: quenches, onset of thermalization?



Boye Buyens, Jutho Haegeman, Florian Hebenstreit, Frank Verstraete, Karel Van Acoleyen, arXiv:1612.00739

… AND QUANTUM FIELD THEORY (1)

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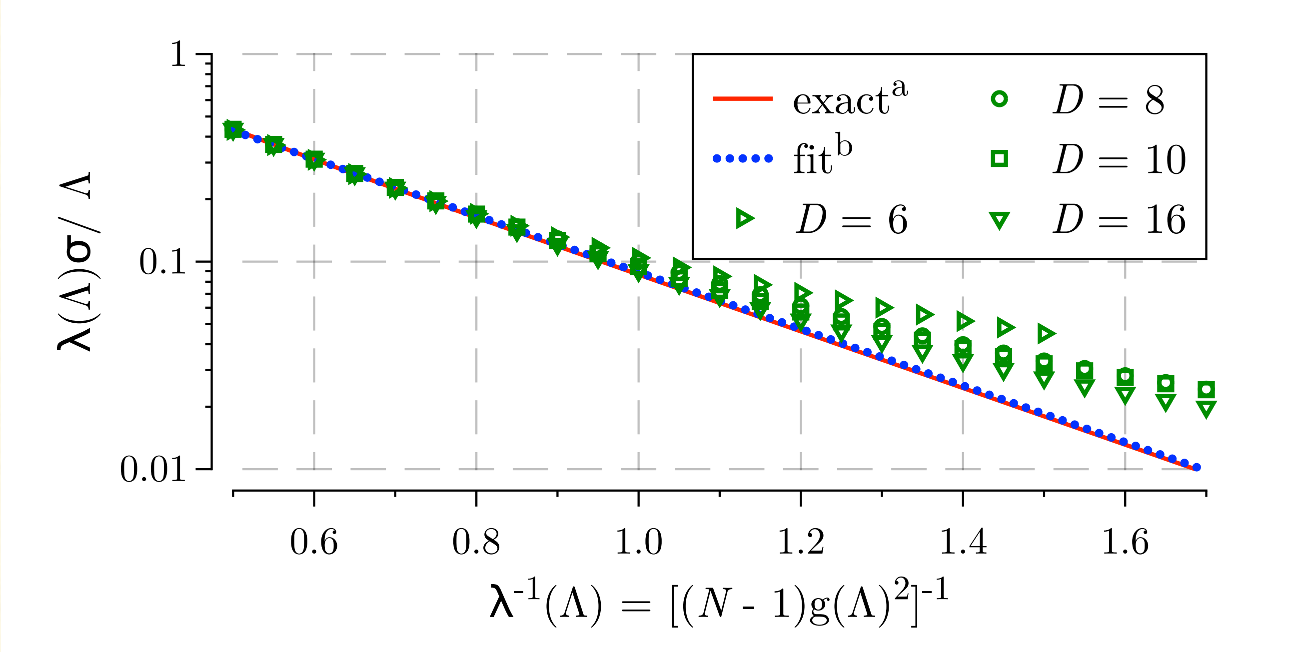
- Very successful for (1+1)d QFT, e.g. Schwinger model
  TMR Byrnes et al, B Buyens et al, MC Bañuls et al, S Montangero et al, …
- Can be extended to (2+1)d (and (3+1)d?):
  first explorations by L Tagliacozzo, E Zohar, …

… AND QUANTUM FIELD THEORY (2)

Tensor networks for continuous systems:

- Continuous MPS: (Verstraete & Cirac, 2010)
  - Chiral condensate in the Gross-Neveu model (N→∞)



- Continuous MERA:
  - So far: only Gaussians
  - Interesting analytical tool to investigate relation with holography

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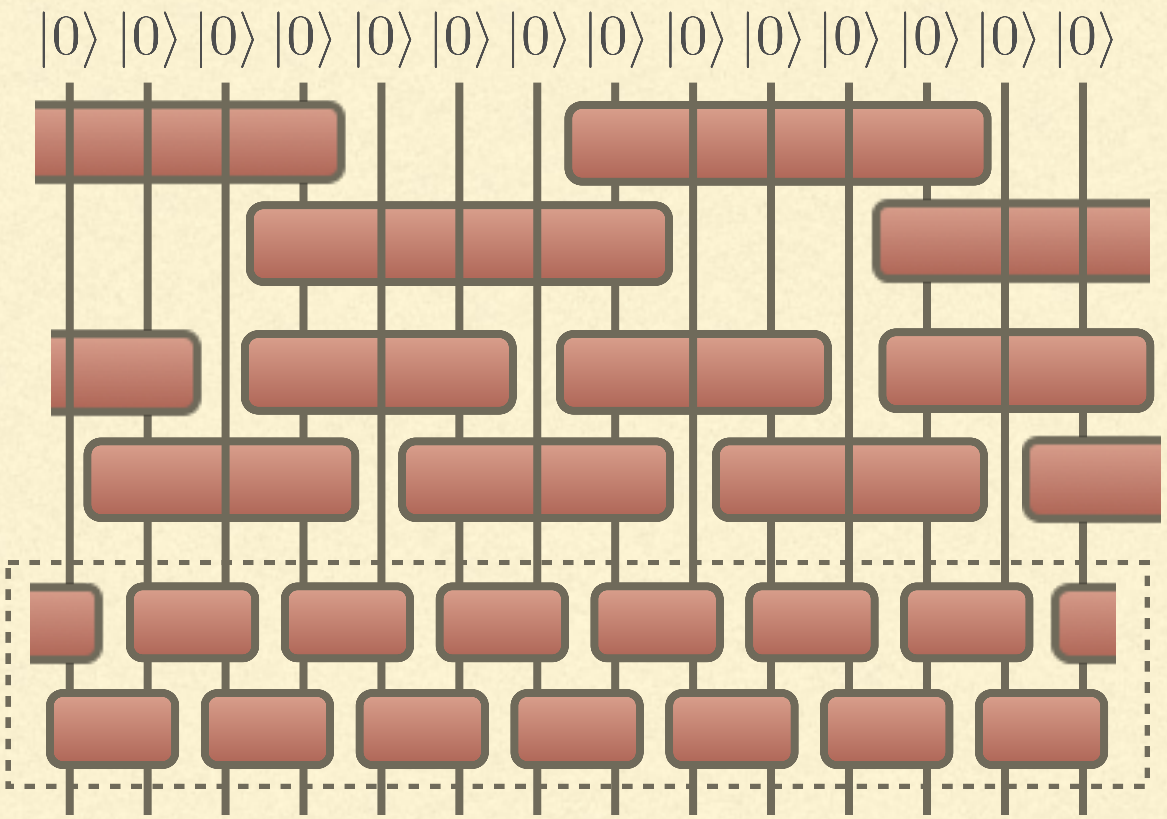
MERA, RENORMALIZATION & WAVELETS

Multiscale entanglement renormalization ansatz (G Vidal)



Coarse-grain quantum state

Disentangle high energy degrees of freedom

Variational ansatz

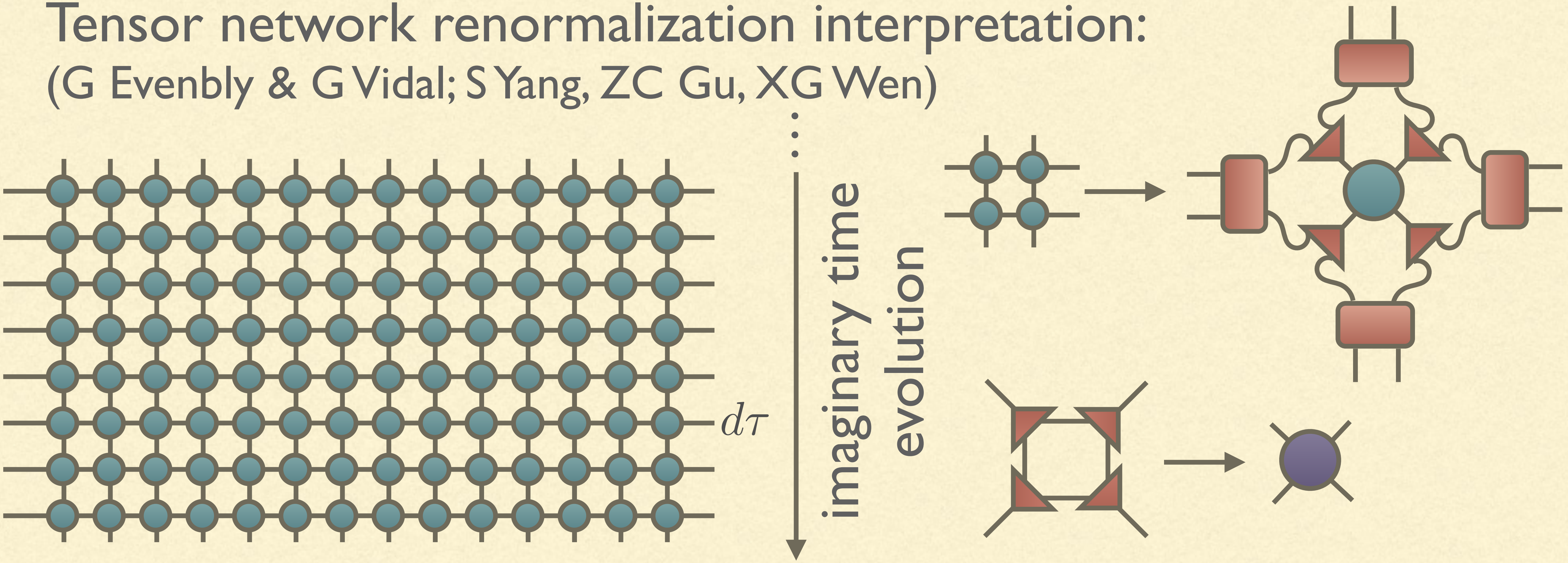
Quantum circuit that prepares state

- Captures power law decay of correlations, logarithmic violation of area law in (1+1)d, …
- Possible relation with holography (AdS/CFT correspondence)

MERA, RENORMALIZATION & WAVELETS

Tensor network renormalization interpretation:

(G Evenbly & G Vidal; S Yang, ZC Gu, XG Wen)



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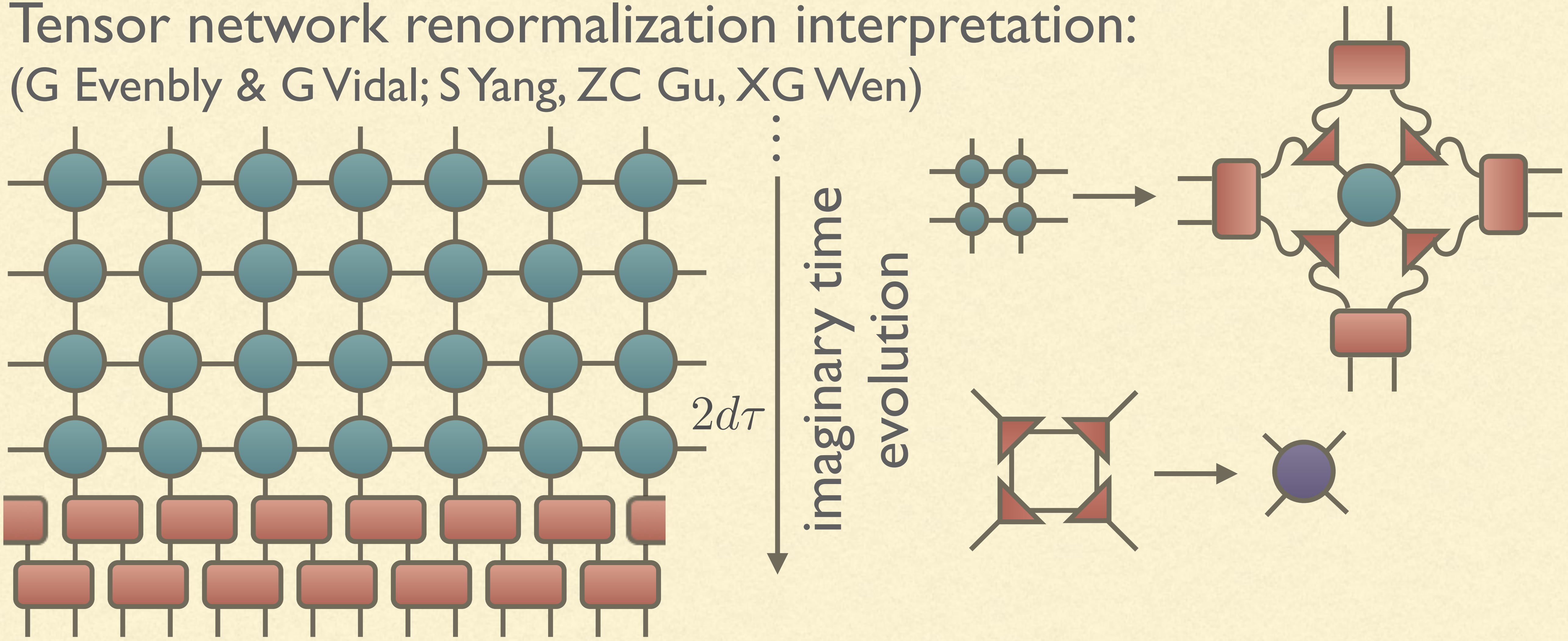

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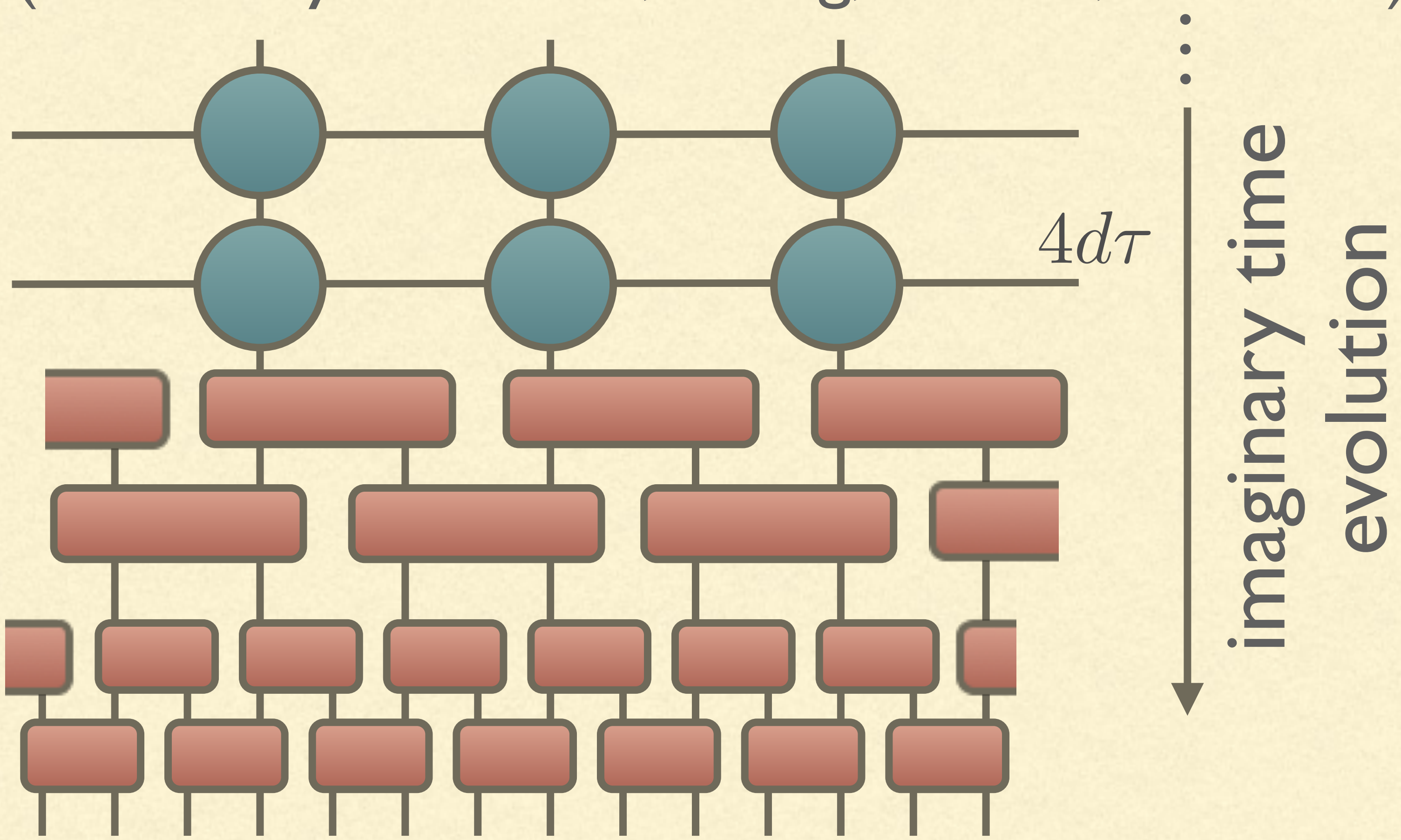

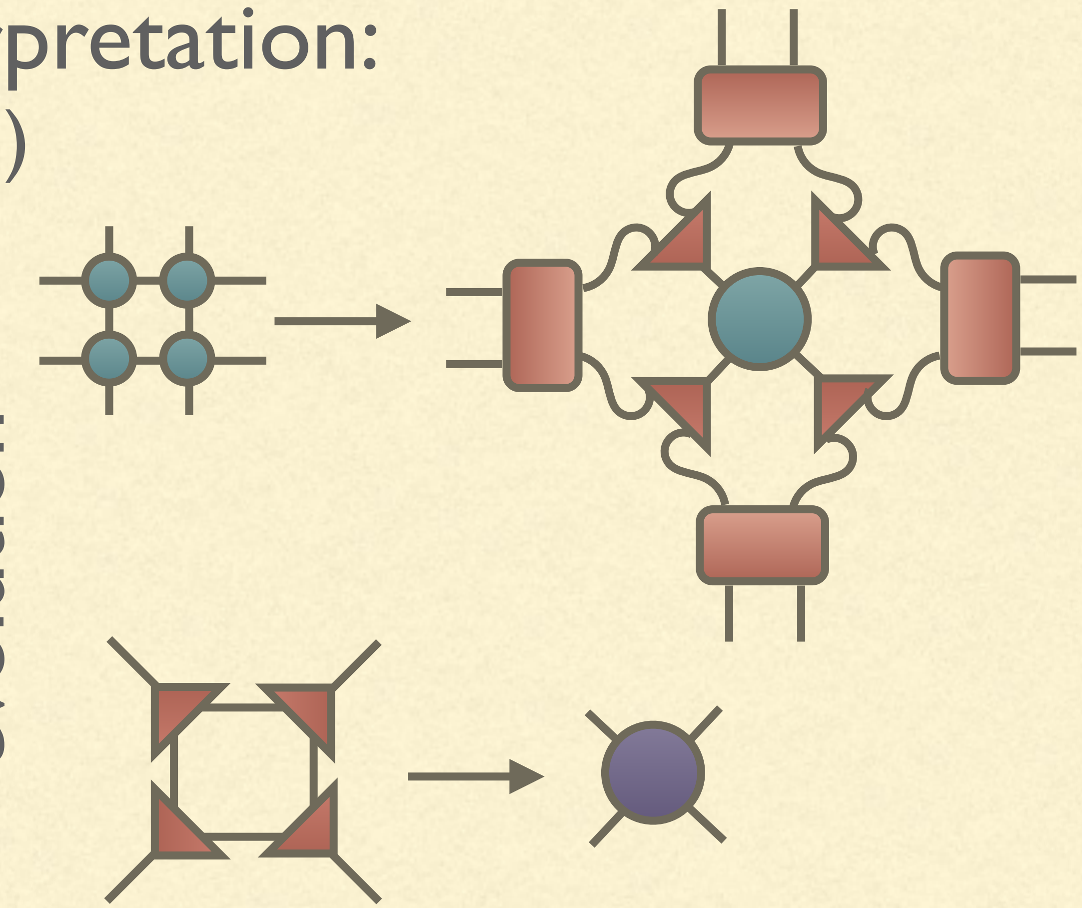

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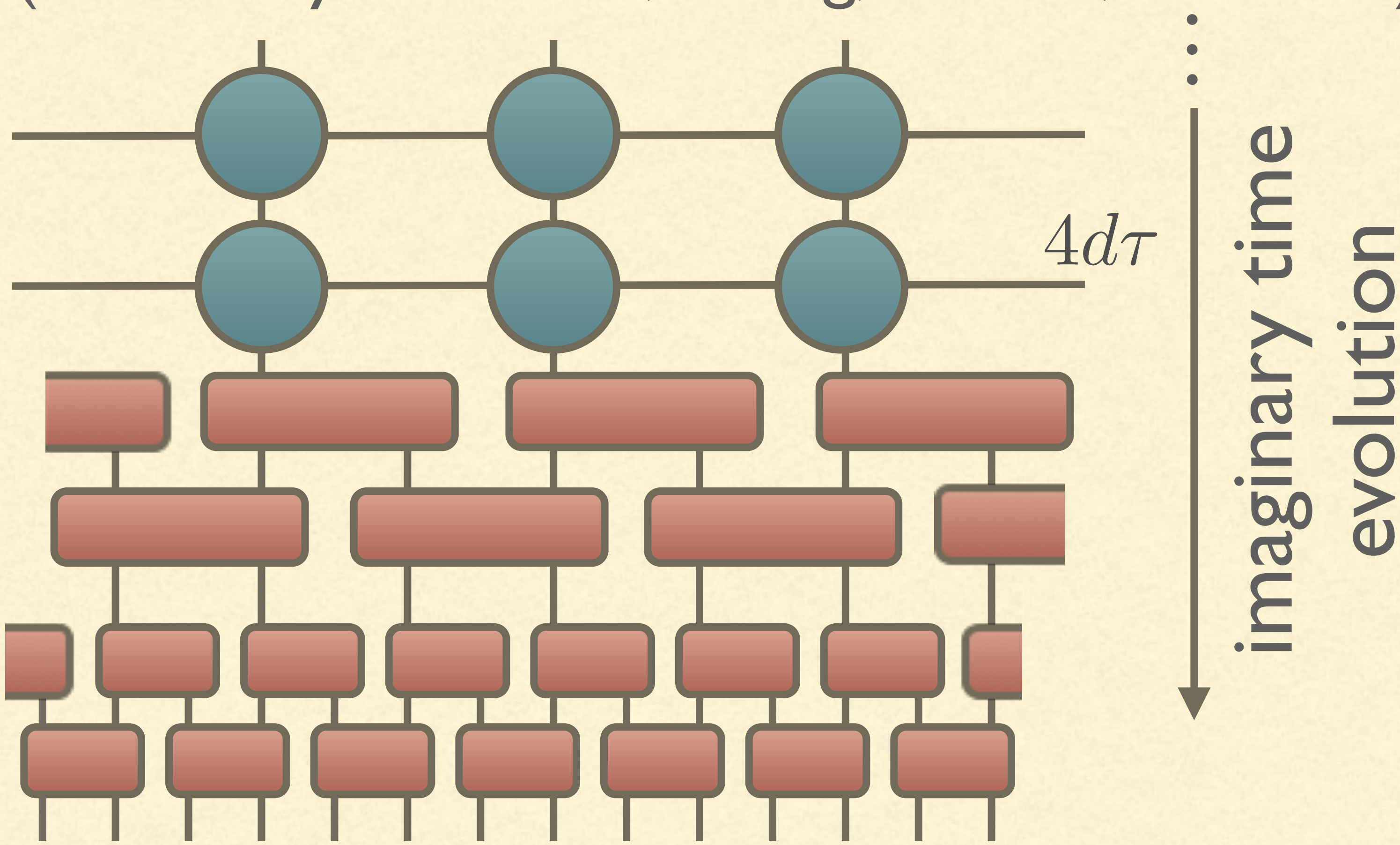
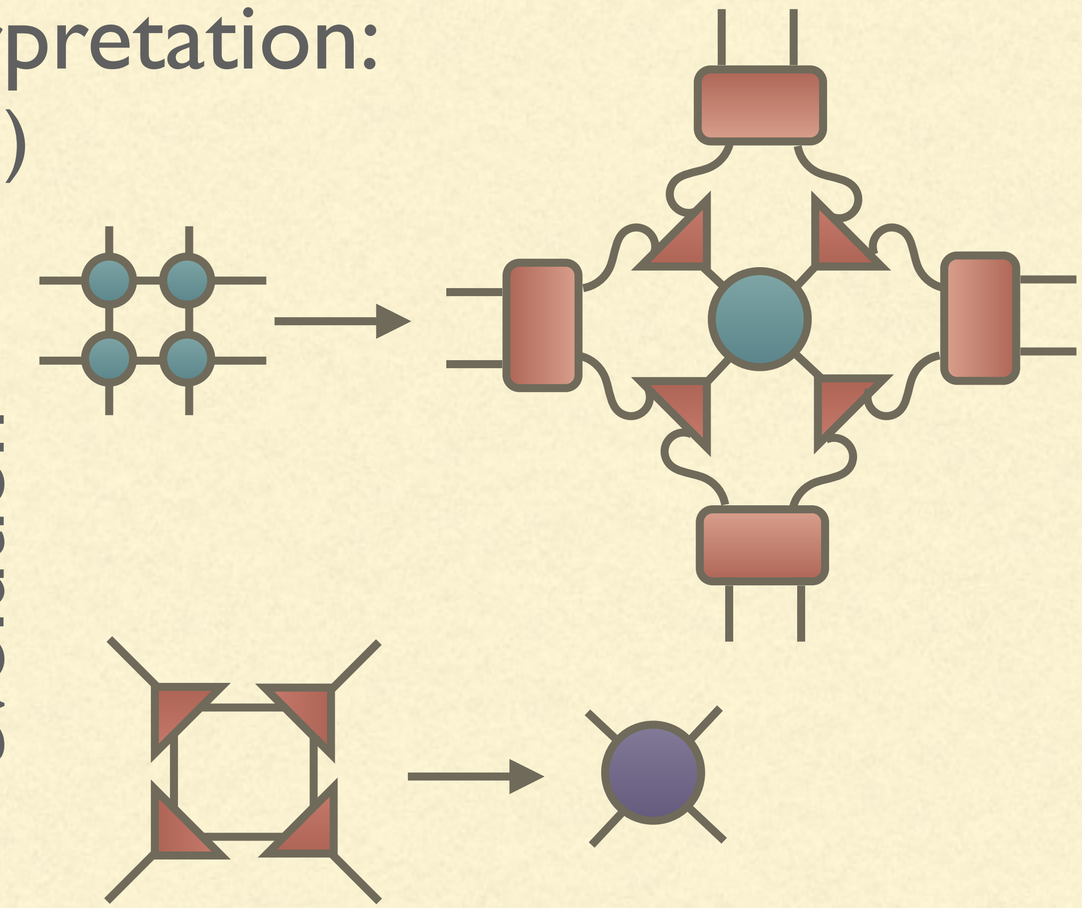




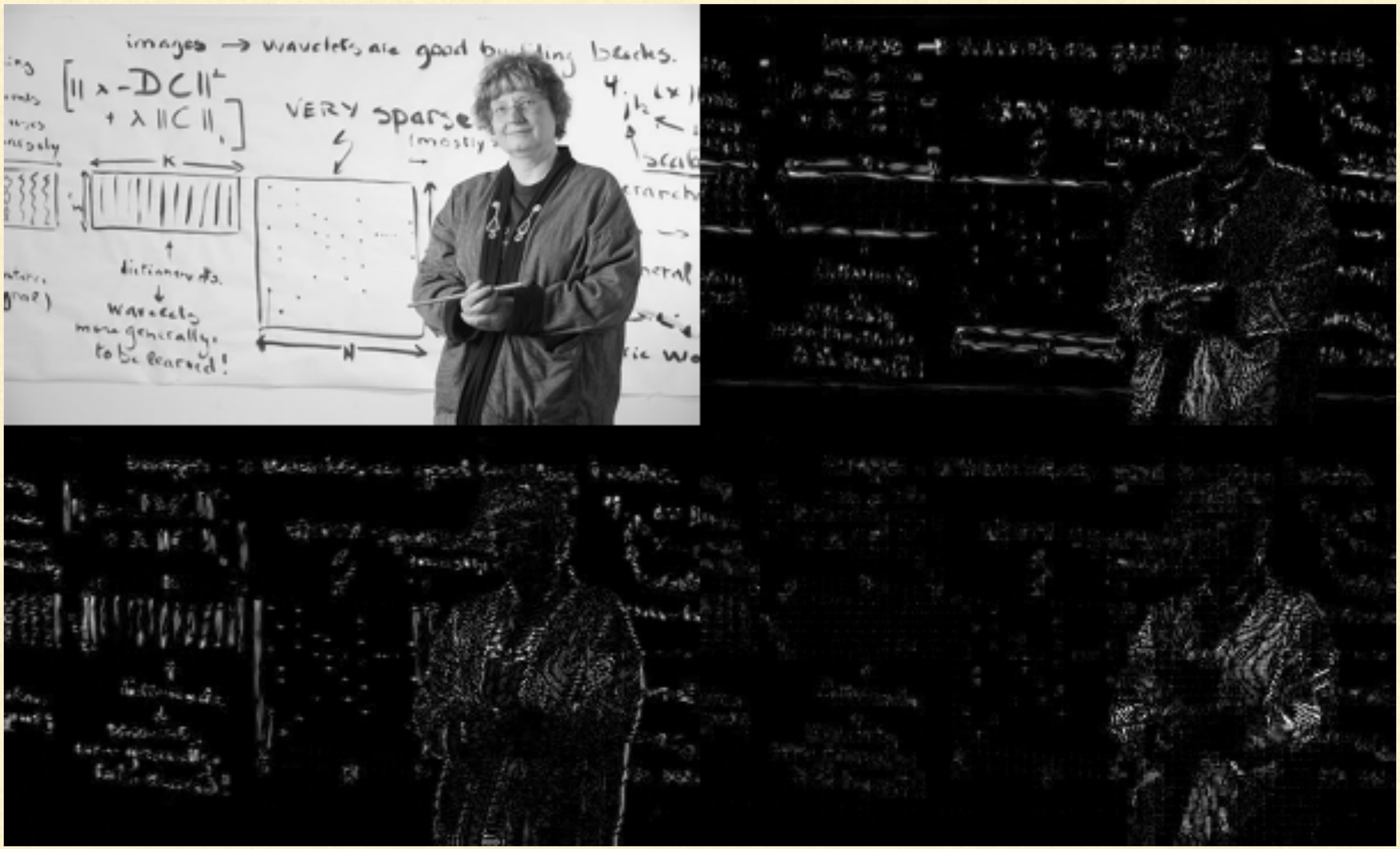
- For classical stat mech systems: can also be done using non-negative matrix factorization → M. Bal et al, PRL 118, 250602 (2017)

MERA, RENORMALIZATION & WAVELETS

Wavelets and renormalization: multiscale analysis

MERA, RENORMALIZATION & WAVELETS

Wavelets and renormalization: multiscale analysis



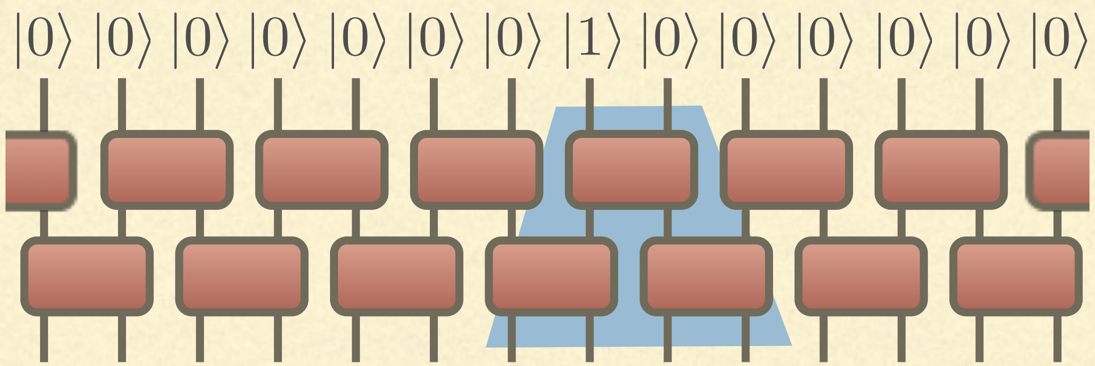
MERA, RENORMALIZATION & WAVELETS

Wavelets and renormalization: multiscale analysis

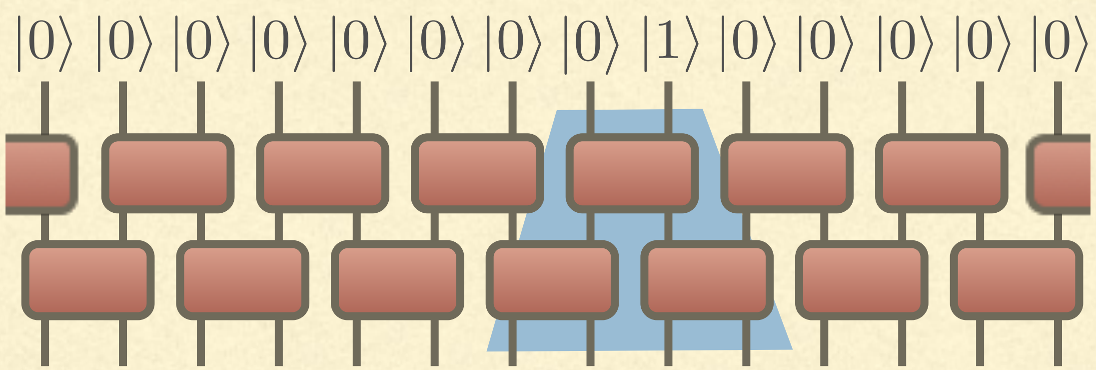
MERA, RENORMALIZATION & WAVELETS

Wavelets and MERA: G Evenbly & S White, PRL 116, 140403 (2016)

A free fermion MERA (unitaries generated by quadratic operators) implements a wavelet transform at the single particle level.



→ scaling coefficients



→ wavelet coefficients

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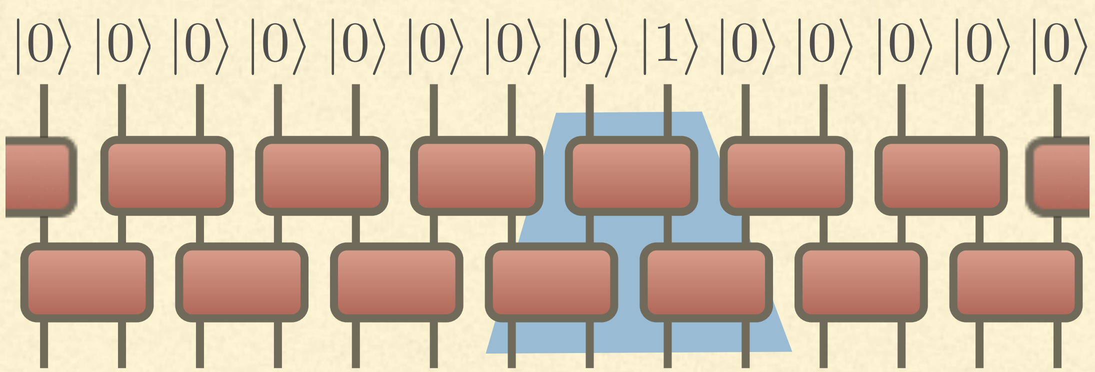


$\longrightarrow$ wavelet coefficients

Free fermion ground state: fill all negative energy modes
→ fill set of modes that span the negative energy subspace (Fermi/Dirac sea)

→ construct wavelets that are completely supported in either positive or negative energy subspace

1+1 DIRAC FERMIONS

Massless Dirac fermions on the lattice: staggering (Kogut-Susskind)

$$H_D = -\sum_n b^\dagger_{1,n} b_{2,n} - b^\dagger_{2,n} b_{1,n+1} + b^\dagger_{2,n} b_{1,n} - b^\dagger_{1,n+1} b_{2,n}$$

$$H_D = \int_{-\pi}^{+\pi} \frac{\mathrm{d}k}{2\pi} \begin{bmatrix} b_1(k) \\ b_2(k) \end{bmatrix}^\dagger \begin{bmatrix} 0 & \mathrm{e}^{-\mathrm{i}k} - 1 \\ \mathrm{e}^{\mathrm{i}k} - 1 & 0 \end{bmatrix} \begin{bmatrix} b_1(k) \\ b_2(k) \end{bmatrix}$$

$$\begin{bmatrix} 0 & \mathrm{e}^{-\mathrm{i}k} - 1 \\ \mathrm{e}^{\mathrm{i}k} - 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ \mathrm{i}\mathrm{e}^{\mathrm{i}k/2} & -\mathrm{i}\mathrm{e}^{\mathrm{i}k/2} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ \mathrm{i}\mathrm{e}^{\mathrm{i}k/2} & -\mathrm{i}\mathrm{e}^{\mathrm{i}k/2} \end{bmatrix} \begin{bmatrix} \sin(k/2) & 0 \\ 0 & -\sin(k/2) \end{bmatrix}, \quad k \in [-\pi, +\pi)$$

$$\begin{bmatrix} 0 & \mathrm{e}^{-\mathrm{i}k} - 1 \\ \mathrm{e}^{\mathrm{i}k} - 1 & 0 \end{bmatrix} u(k) = u(k) \begin{bmatrix} -|\sin(k/2)| & 0 \\ 0 & |\sin(k/2)| \end{bmatrix}, \quad k \in [-\pi, +\pi)$$

$$u(k) = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -\mathrm{i}\,\mathrm{sign}(k)\mathrm{e}^{\mathrm{i}k/2} & \mathrm{i}\,\mathrm{sign}(k)\mathrm{e}^{\mathrm{i}k/2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -\mathrm{i}\,\mathrm{sign}(k)\mathrm{e}^{\mathrm{i}k/2} \end{bmatrix} \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

1+1 DIRAC FERMIONS

$$u(k) = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ -\mathrm{isign}(k)\mathrm{e}^{\mathrm{i}k/2} & \mathrm{isign}(k)\mathrm{e}^{\mathrm{i}k/2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -\mathrm{isign}(k)\mathrm{e}^{\mathrm{i}k/2} \end{bmatrix} \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

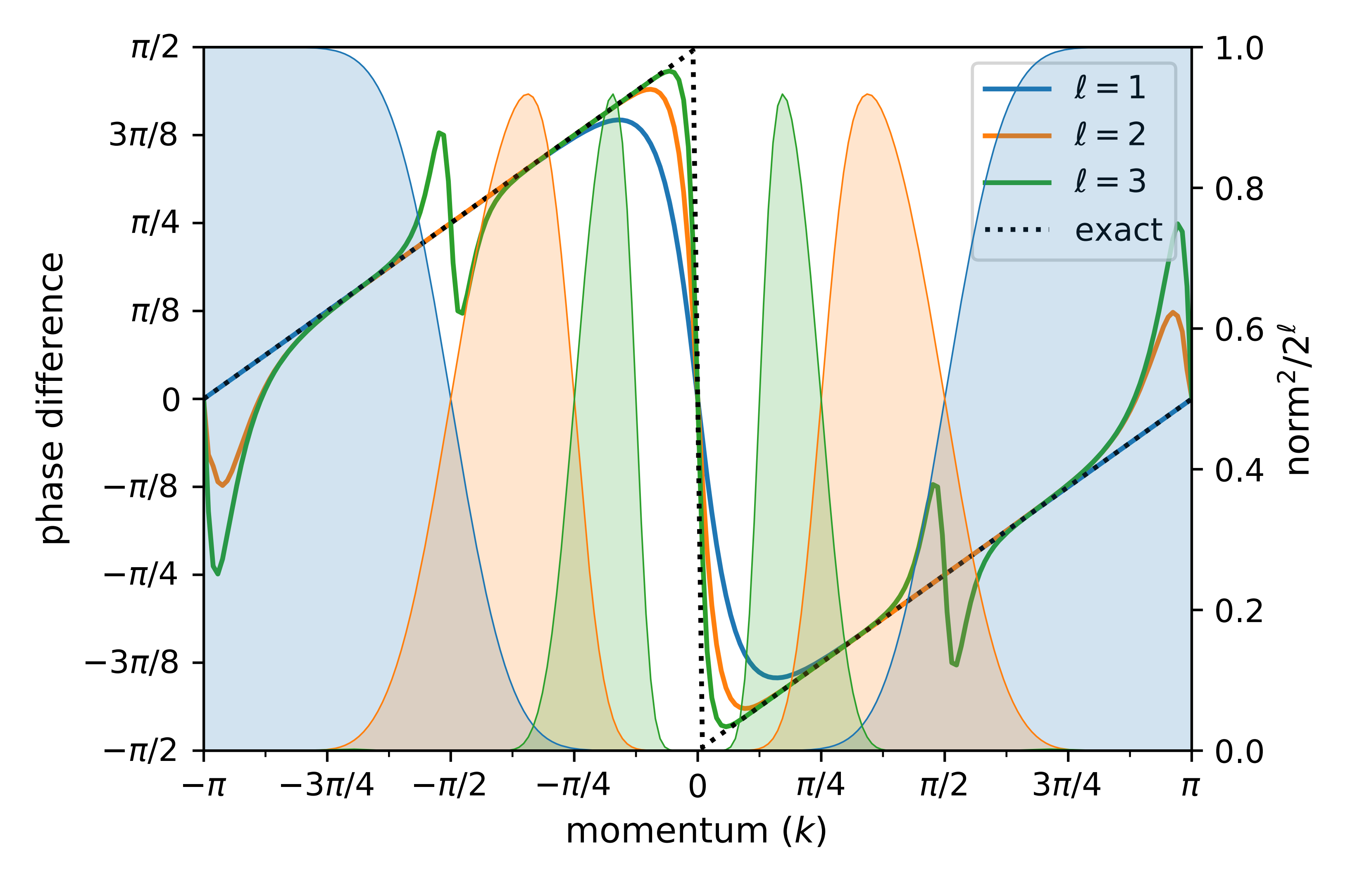
A pair of wavelet transforms such that wavelet filters in Fourier domain $(g_w(k), h_w(k))$ have equal magnitude and a relative phase difference $-\mathrm{isign}(k)\mathrm{e}^{\mathrm{i}k/2}$.

Scaling filters $(g_s(k), h_s(k))$ should have phase difference $\mathrm{e}^{\mathrm{i}k/2}$ (half shift or half delay condition) → same phase difference for wavelets from higher levels of the transform (scale invariance).

Impossible with filters of finite support ⇒ approximation?

1+1 DIRAC FERMIONS

Problem considered by Selesnick et al: a family of solutions, satisfying $h_s(k) = \mathrm{e}^{\mathrm{i}\theta(k)} g_s(k),$ in terms of two parameters K and L, leading to filters of width 2(K+L):



K = 4

L = 6

FERMI SURFACES

Non-relativistic fermions hopping at half filling:

$$H_1 = -\sum_{n\in\mathbb{Z}} a_n^\dagger a_{n+1} + a_{n+1}^\dagger a_n \xrightarrow{b_{1,n} = (-1)^n a_{2n},\, b_{2,n} = (-1)^n a_{2n+1}} H_D$$

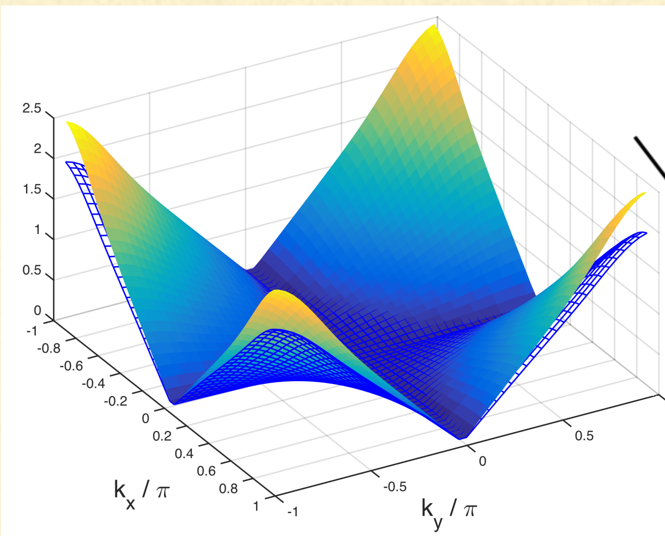
$$H_2 = -\sum_{(m,n)\in\mathbb{Z}^2} a_{m,n}^\dagger a_{m+1,n} + a_{m+1,n}^\dagger a_{m,n} + a_{m,n}^\dagger a_{m,n+1} + a_{m,n+1}^\dagger a_{m,n}$$

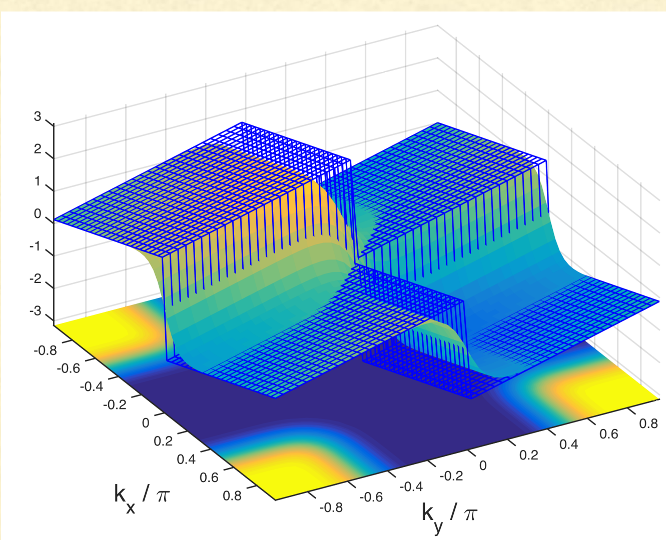
$$\Big\downarrow\ b_{1,x,y} = (-1)^{x+y} a_{x+y,x-y},\, b_{2,x,y} = (-1)^{x+y} a_{x+y+1,x-y}$$

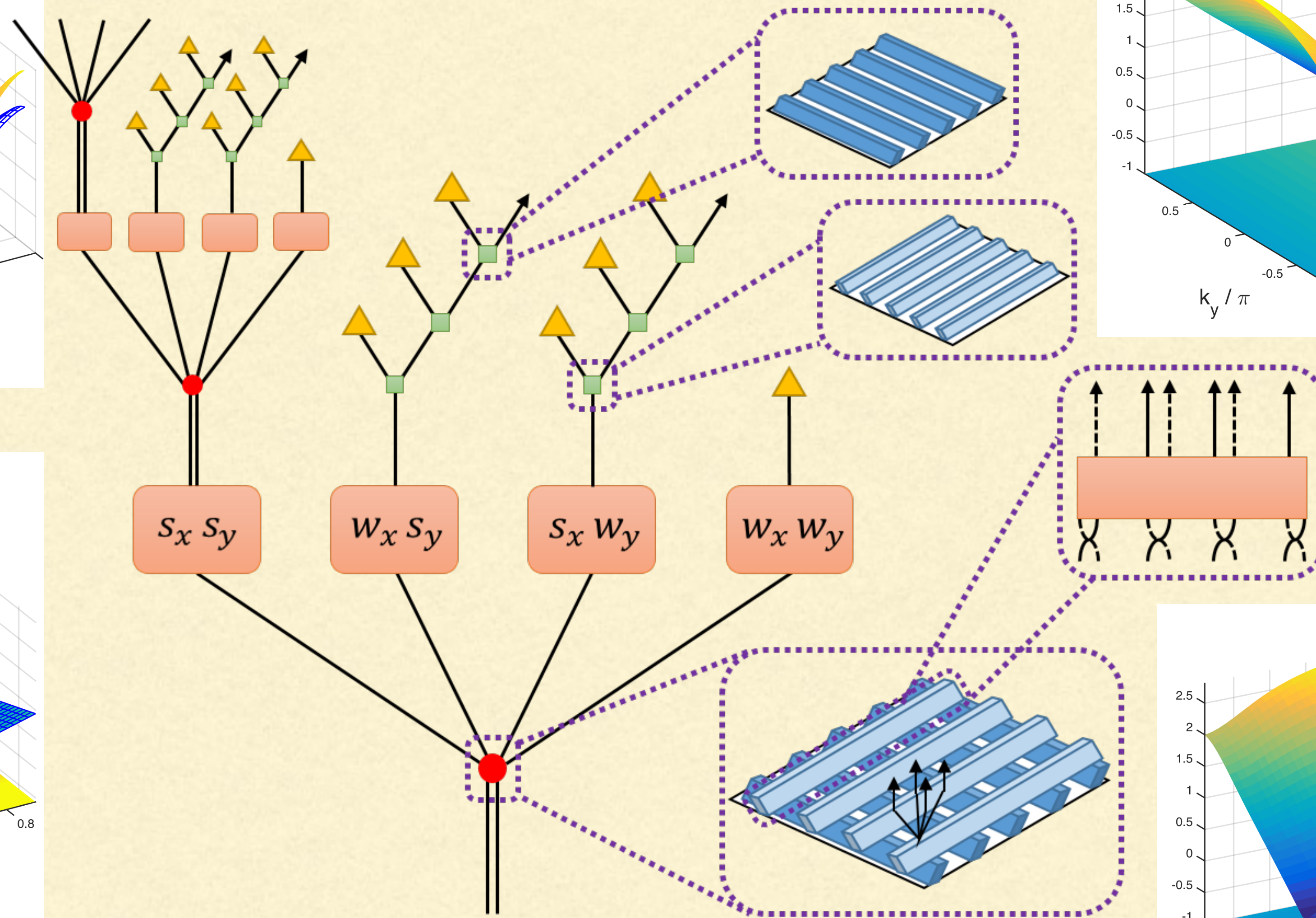
$$H = \int_{[-\pi,\pi)^2} \mathrm{d}k_x \mathrm{d}k_y \begin{bmatrix} b_1(k_x,k_y) \\ b_2(k_x,k_y) \end{bmatrix}^\dagger \begin{bmatrix} 0 & (1-\mathrm{e}^{-\mathrm{i}k_x})(1-\mathrm{e}^{-\mathrm{i}k_y}) \\ (1-\mathrm{e}^{\mathrm{i}k_x})(1-\mathrm{e}^{\mathrm{i}k_y}) & 0 \end{bmatrix} \begin{bmatrix} b_1(k_x,k_y) \\ b_2(k_x,k_y) \end{bmatrix}$$

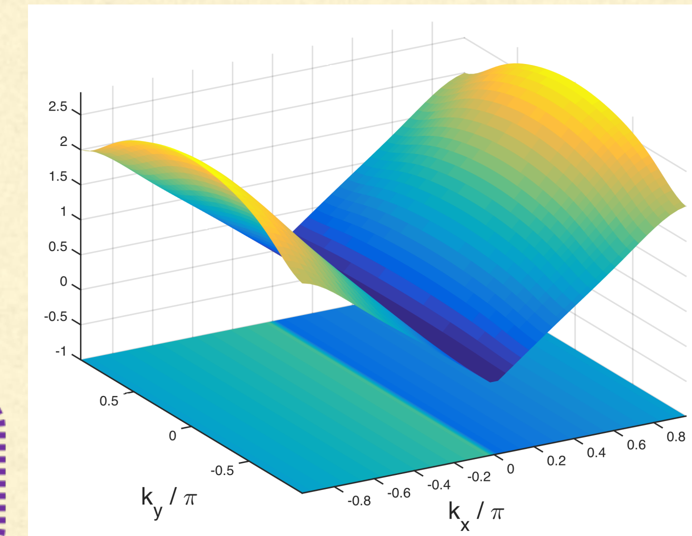
$$u(k_x,k_y) = \begin{bmatrix} 1 & 0 \\ 0 & -\mathrm{i}\,\mathrm{sign}(k_x)\mathrm{e}^{\mathrm{i}k_x/2} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -\mathrm{i}\,\mathrm{sign}(k_y)\mathrm{e}^{\mathrm{i}k_y/2} \end{bmatrix} \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

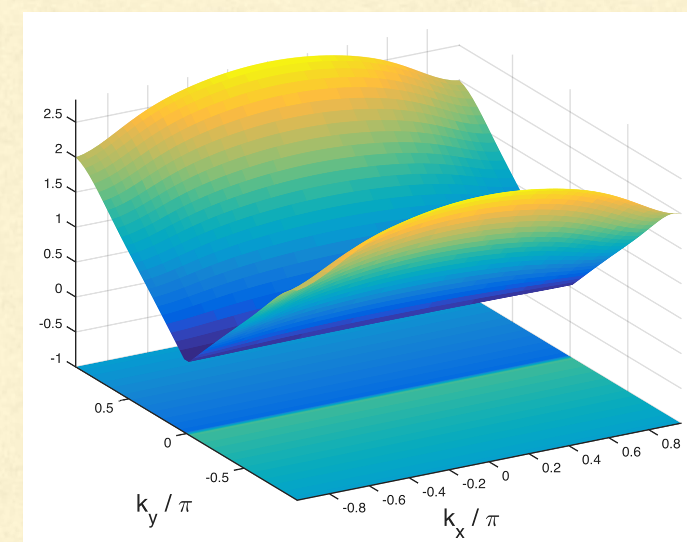
FERMI SURFACES











Branching MERA (Evenbly & Vidal)

R Shankar, RG approach to interacting fermions (RMP 66, 129)

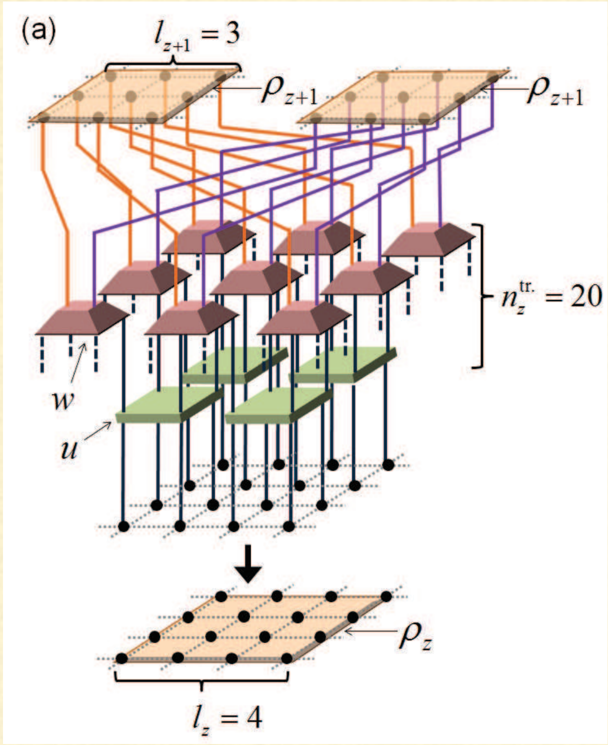
FERMI SURFACES

$$S_{\text{1d MERA}}(R) \leq c_1 + S_{\text{1d MERA}}(R/2) \leq \ldots \leq c_1 \log_2(R) + \mathcal{O}(1)$$

$$S_{\text{2d MERA}}(R) \leq c_2 R + S_{\text{2d MERA}}(R/2) \leq \ldots \leq 2c_2 R + \mathcal{O}(1)$$

$$\begin{aligned} S(R) &\leq c_2 R + 2(R/2) S_{\text{1d MERA}}(R/2) + S(R/2) \\ &\leq \ldots \\ &\leq 2c_1 R \log_2 R + \mathcal{O}(R) \end{aligned}$$

$$[\tilde{S}(R) \leq c_2 R + 2\tilde{S}(R/2) \leq \cdots \leq c_2 R \log_2(R) + \mathcal{O}(R)]$$

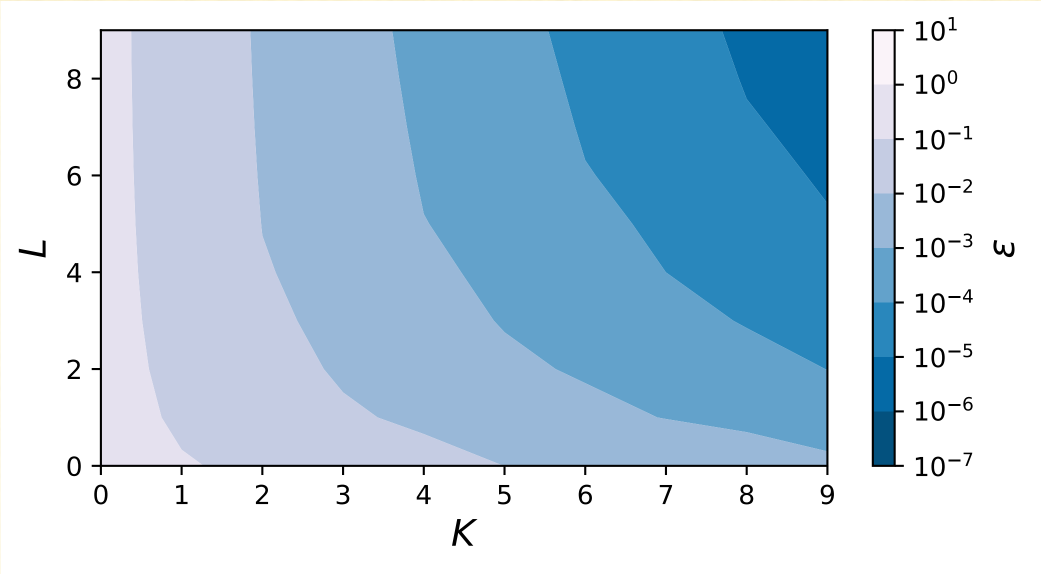


RIGOROUS APPROXIMATION RESULT

Given pair of scaling filters:

$$|h_s(k) - \mathrm{e}^{\mathrm{i}k/2} g_s(k)| \leq \epsilon, \quad \forall k \in [-\pi, +\pi)$$

$$B = \|\phi_s\|_\infty$$



Let: $f \in \ell_2(\mathbb{Z}) : a(f) = \sum_{n \in \mathbb{Z}} f[n] a_n$

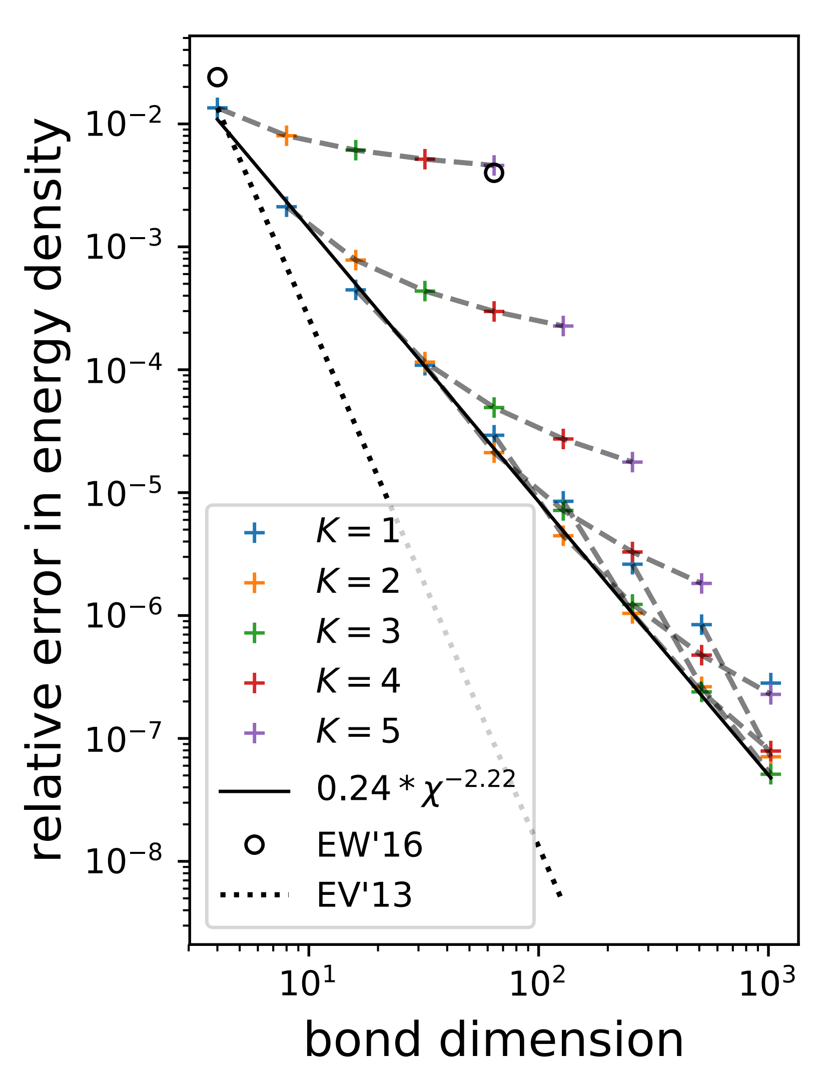
$$G(\{f_i\})_\Psi = \langle \Psi | a^\dagger(f_1) \ldots a^\dagger(f_N) a_{(} f_{N+1}) \ldots a(f_{2N}) | \Psi \rangle$$

Then: $|G(\{f_i\})_\Omega - G(\{f_i\})_{\Omega_{\mathrm{MERA}}}| \leq 24\sqrt{N}\sqrt{C 2^{-\mathcal{L}/2} + 6\epsilon(\log_2(C/\epsilon))^2}$

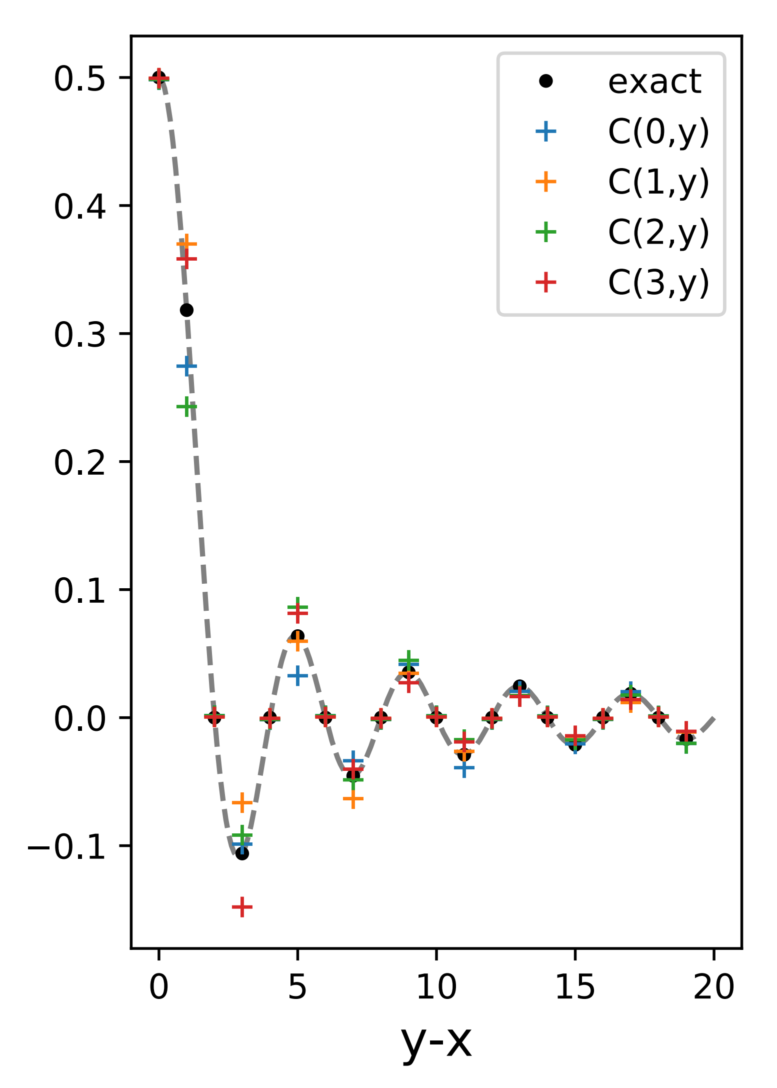
Where: $C = 2^{3/2}\sqrt{D(\{f_i\})} B(K+L)$

$\mathcal{L}$: number of layers in the MERA

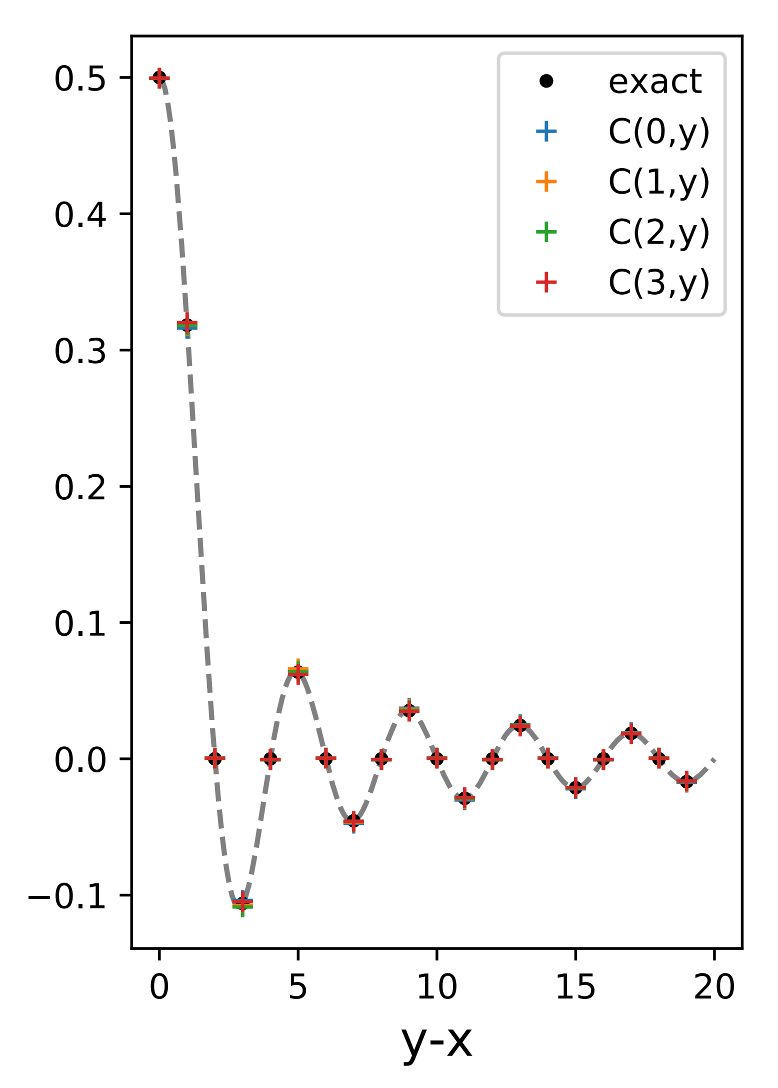
RIGOROUS APPROXIMATION RESULT



K=L=1



K=L=3



OUTLOOK AND EXTENSIONS

- Extensions
  - Massive theories
  - Pairing term:
    fermion number → fermion parity conservation
- Outlook
  - Dirac cones, topological insulators,…?
  - Relevance for interacting theories?
    (e.g. with asymptotic freedom)

QUESTIONS?