BQP-completeness of Scattering in Quantum Field Theory

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[ArXiv:1703.00454]

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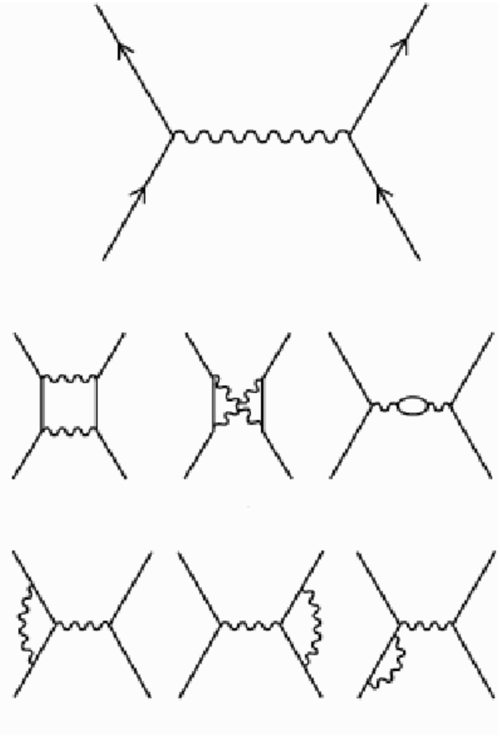
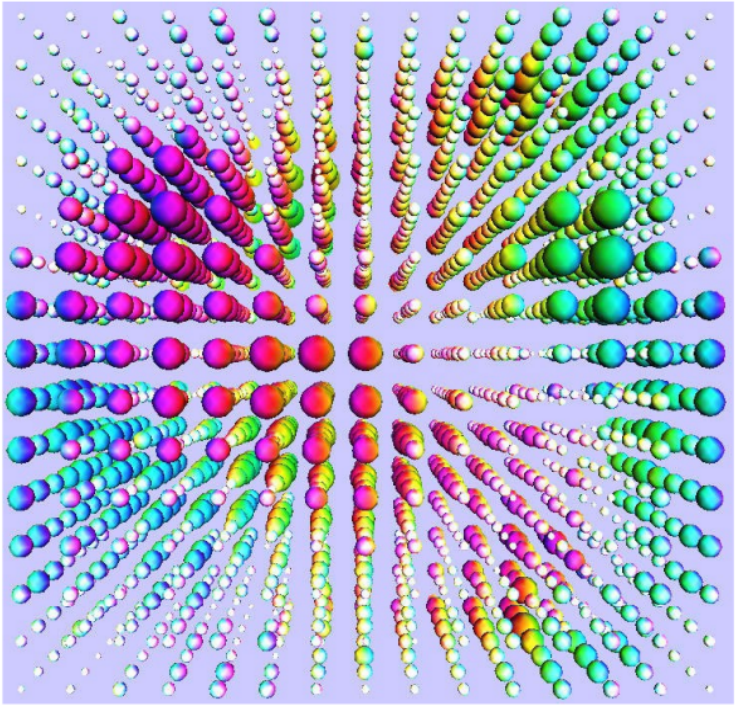
QFT Seems Hard Classically

Feynman diagrams



Break down at strong coupling or high precision

Lattice methods



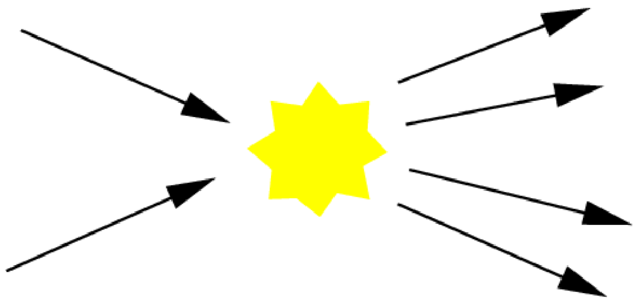
Cannot calculate scattering amplitudes

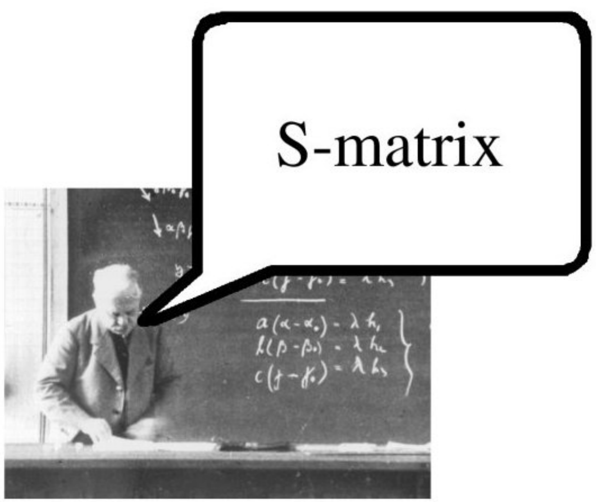
For some parameters we have no poly-time classical algorithm.

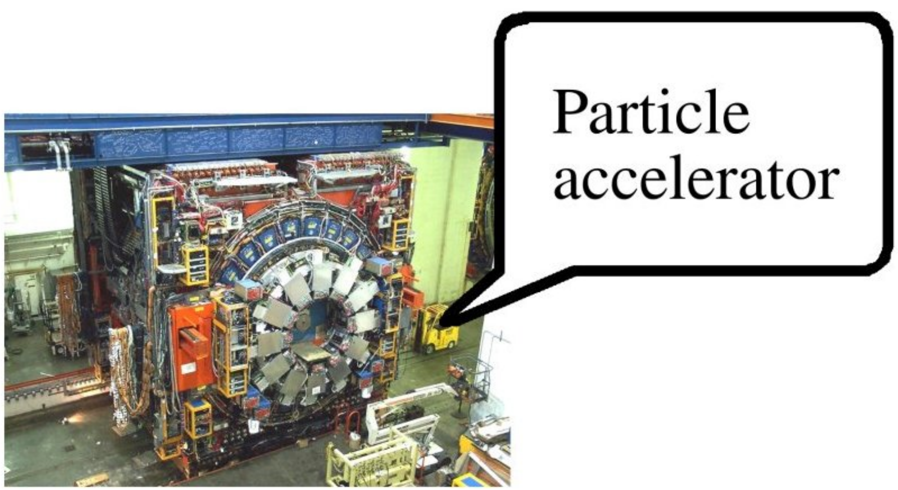
A QFT Computational Problem

Input: a list of momenta of incoming particles.

Output: a list of momenta of outgoing particles.





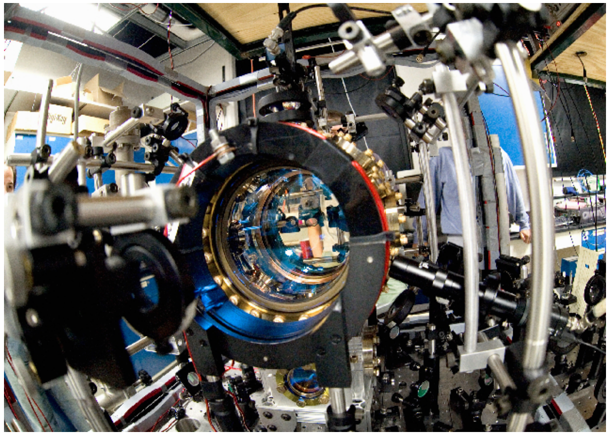


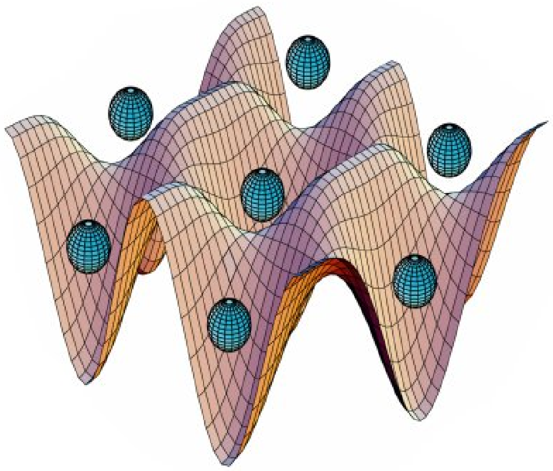
Quantum Algorithms

- Efficient quantum simulation algorithms:

	Bosonic	Fermionic
Massive	Jordan, Lee, Preskill *Science*, 336:1130 (2012)	Jordan, Lee, Preskill *ArXiv:1404.7115* (2014)
Massless	TODO	TODO

- Analog simulations:





How hard is it and when?

- Things that might make QFT hard:
  - Fermionic sign problem
  - Fermion doubling problem
  - Strong coupling
  - High-dimensional spacetime (D=4)
  - Massless particles
  - “Critical slowing down” at quantum phase transitions
  - Large # particles

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- Fermionic sign problem
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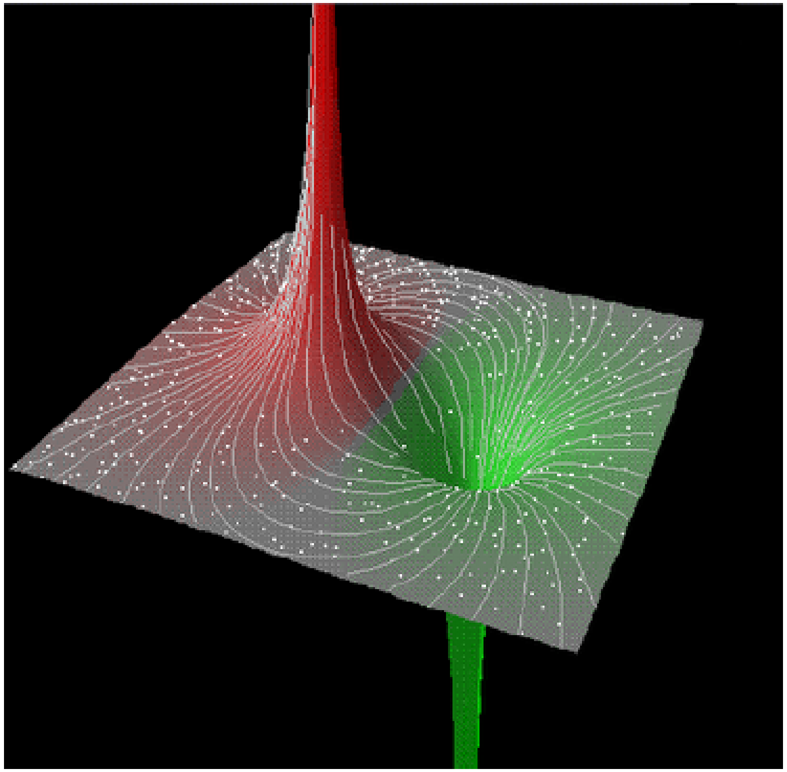
Our result: Large # particles is all it takes to obtain hard problems, provided we allow external fields.

Quantum Fields



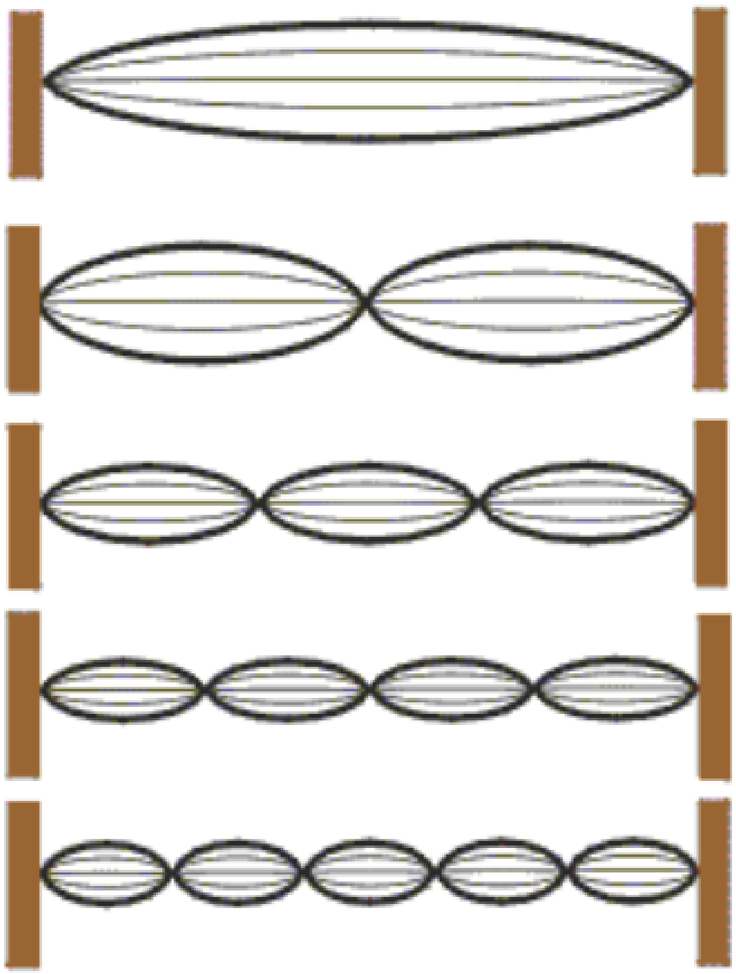
A classical field is described by its value at every point in space.

$$E(r) = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$

A quantum field is a superposition of classical field configurations.

$$|\Psi\rangle = \int \mathcal{D}[E]\Psi[E]\;|E\rangle$$

Particles Emerge from Fields



Particles of different energy are different resonant excitations of the field.

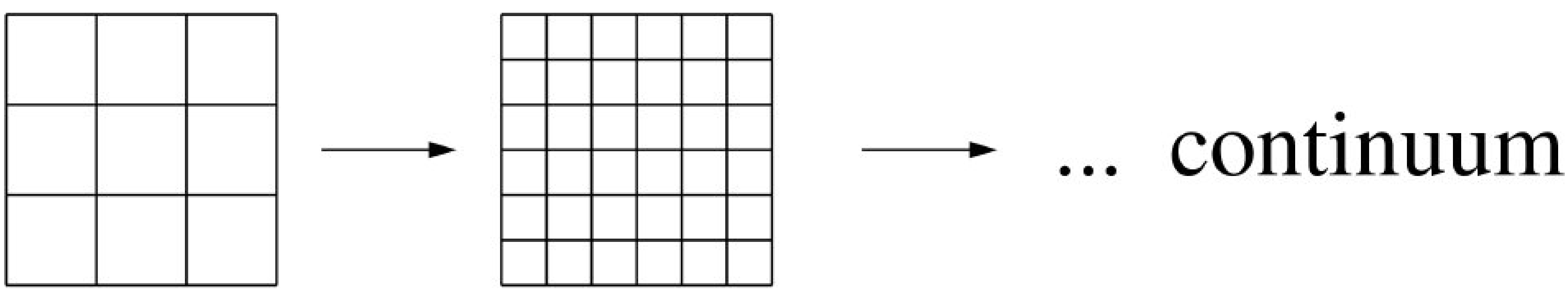
Phi-Fourth Theory: Constructively

- Make a lattice of anharmonic oscillators:

$$[\phi_j, \pi_k] = \frac{i}{a}\delta_{jk}\mathbb{1}$$

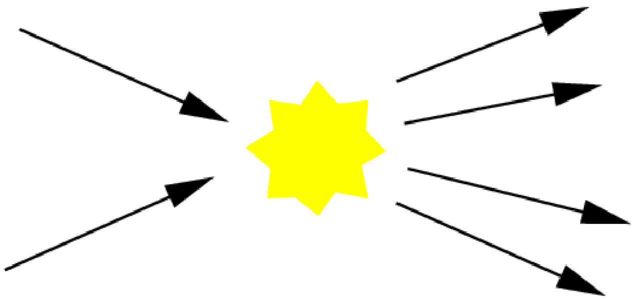
$$H = \sum_j a\left[\frac{1}{2}\pi_j^2 + \frac{m_0}{2}\phi_j^2 + \left(\frac{\phi_{j+1} - \phi_j}{a}\right)^2 + \frac{\lambda_0}{4!}\phi_j^4\right]$$

- Take limit of $a \rightarrow 0$



What do field theorists want to calculate?

– Scattering amplitudes:



– Correlation functions: $\langle \mathrm{vac}|\phi(x)\phi(y)|\mathrm{vac}\rangle$

– Partition function:

$$Z[J] = \langle \mathrm{vac}|T\left\{\exp\left[-i\int d^D x\,(H + J(x)\phi(x))\right]\right\}|\mathrm{vac}\rangle$$

$$Z[J] = \langle \text{vac}|T\left\{\exp\left[-i\int d^D x\,(H + J(x)\phi(x))\right]\right\}|\text{vac}\rangle$$

$$\frac{\delta}{\delta J(x)} Z = \langle \phi(x) \rangle$$

$$\frac{\delta}{\delta J(x)} \frac{\delta}{\delta J(y)} Z = \langle \phi(x)\phi(y) \rangle$$

$$\vdots$$

Partition functions yield correlation functions.

LSZ Formula

$$\prod_1^n \int d^4x_i \, e^{ip_i\cdot x_i} \prod_1^m \int d^4y_j \, e^{-ik_j\cdot y_j} \, \langle\Omega| \, T\{\phi(x_1)\cdots\phi(x_n)\phi(y_1)\cdots\phi(y_m)\} \, |\Omega\rangle$$

$$\underset{\substack{\text{each } p_i^0 \to +E_{\mathbf{p}_i} \\ \text{each } k_j^0 \to +E_{\mathbf{k}_j}}}{\sim} \left(\prod_{i=1}^n \frac{\sqrt{Z}\, i}{p_i^2 - m^2 + i\epsilon}\right)\left(\prod_{j=1}^m \frac{\sqrt{Z}\, i}{k_j^2 - m^2 + i\epsilon}\right) \langle \mathbf{p}_1 \cdots \mathbf{p}_n | \, S \, | \mathbf{k}_1 \cdots \mathbf{k}_m \rangle$$

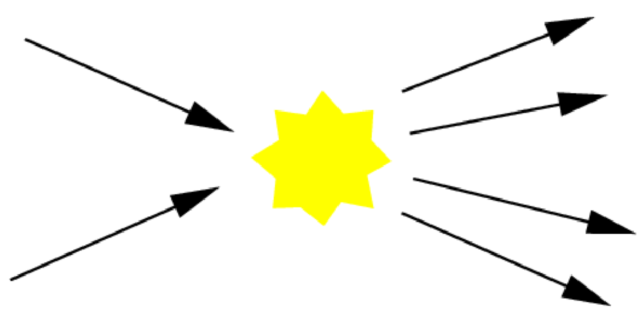
Correlation functions yield scattering amplitudes.

$$Z[J]$$

Functional Derivatives

$$\langle \mathrm{vac}|\phi(x)\phi(y)|\mathrm{vac}\rangle$$

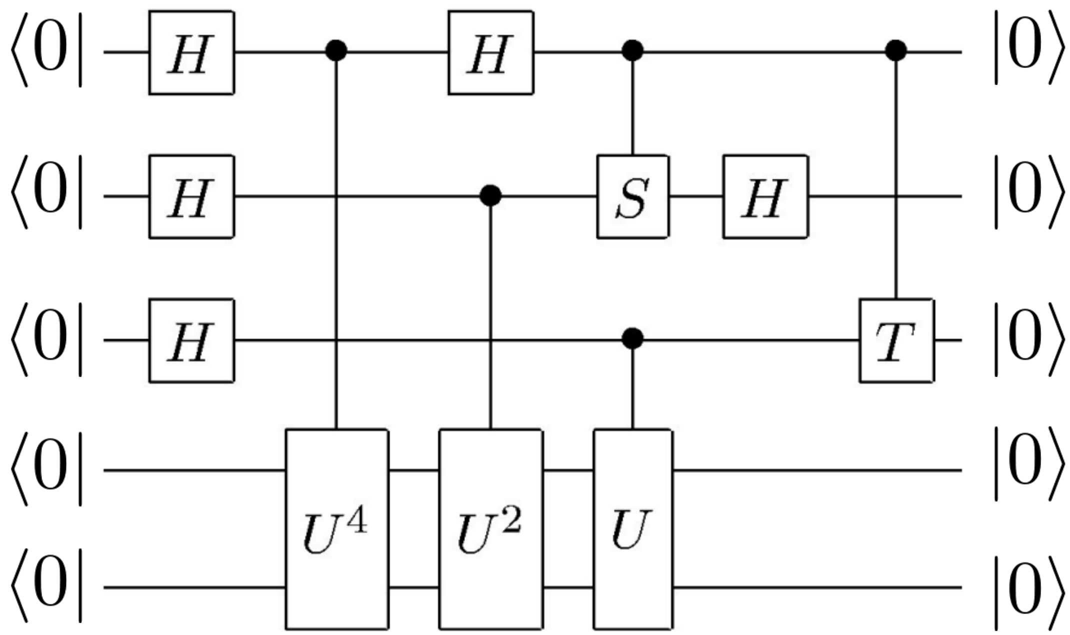
Fourier Transform (LSZ)



Z[J] also directly answers a physics question:

If we apply external field J(x,t), what is the amplitude to remain in vacuum state?

Our Reduction



Choice of circuit

$$Z[J] = \langle \mathrm{vac} | \ldots | \mathrm{vac} \rangle$$

Choice of J(x,t)

Non-Relativistic Limit

$$H[J_2] = H_{\phi^4} + \int d^d x \, J_2(x) \phi(x)^2$$

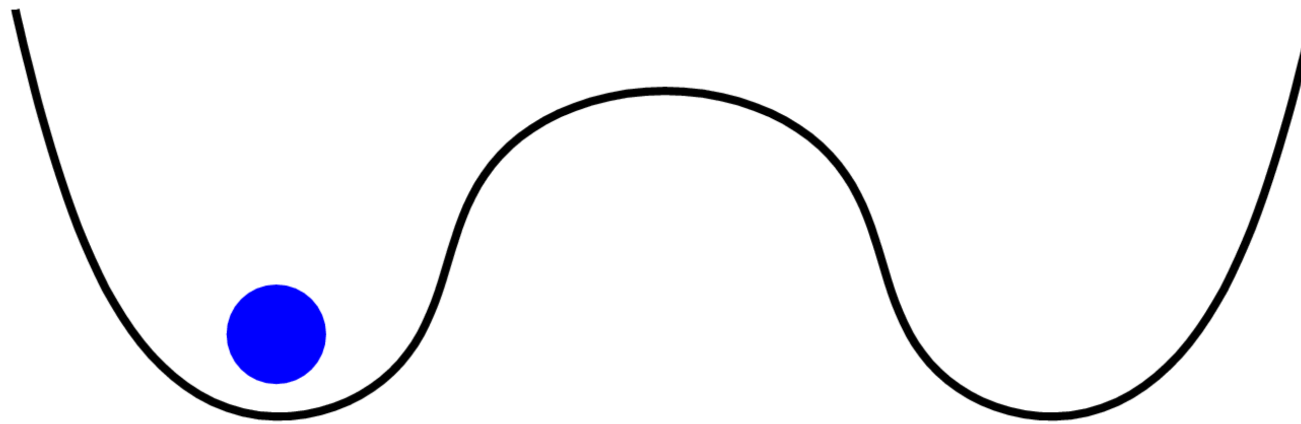
Low energy,

Fixed particle #

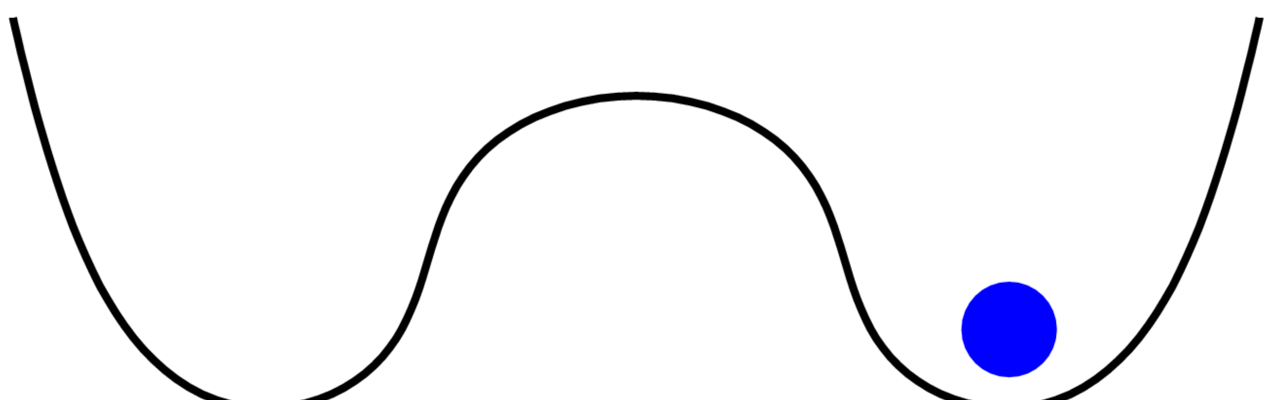
$$H_{\mathrm{NR}} = \sum_j \left(-\frac{p_j^2}{2m} + \frac{1}{m} J_2(x_j) \right) + O(\lambda)$$

Double-Well Qubits

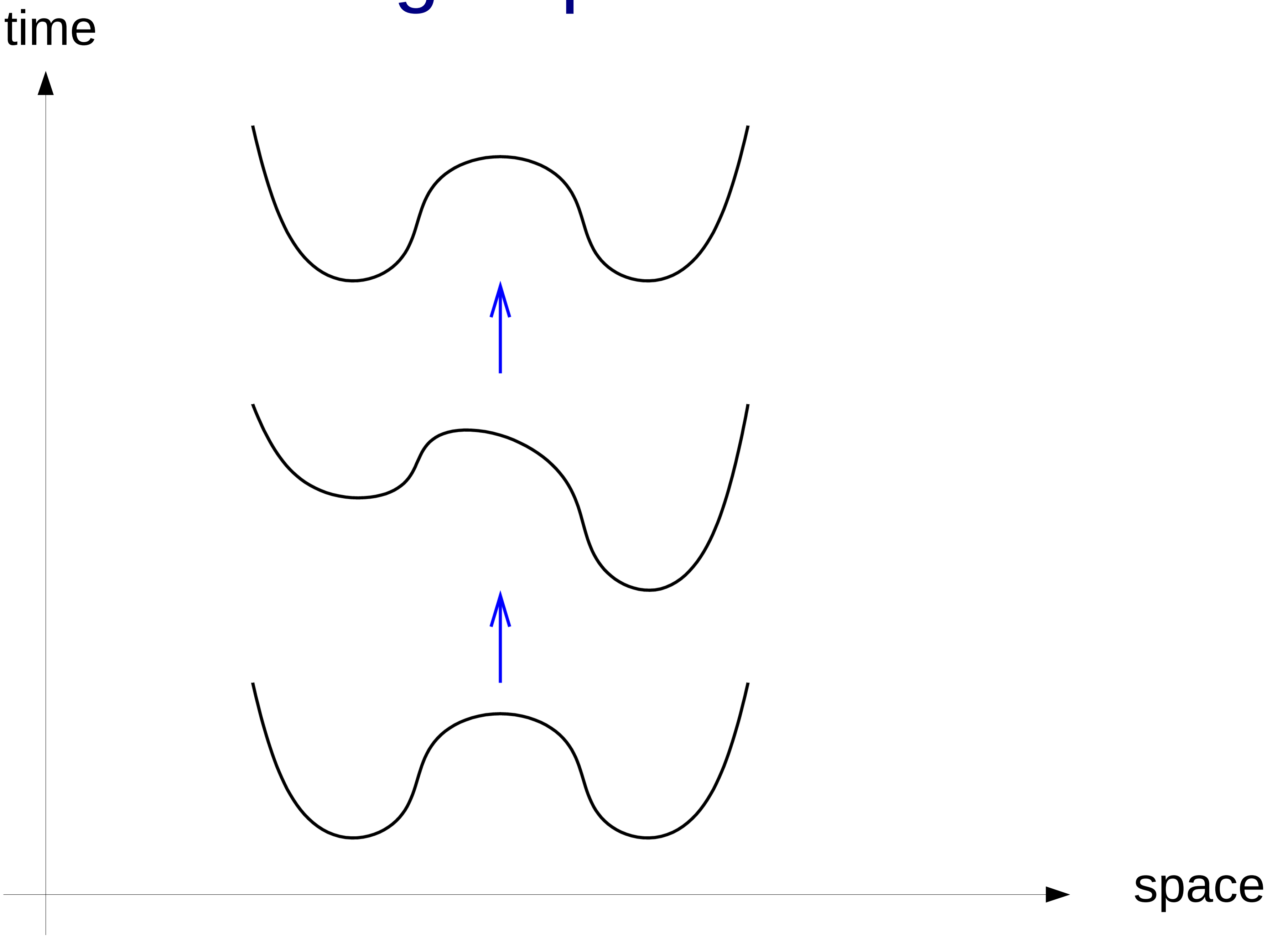
$|0\rangle$



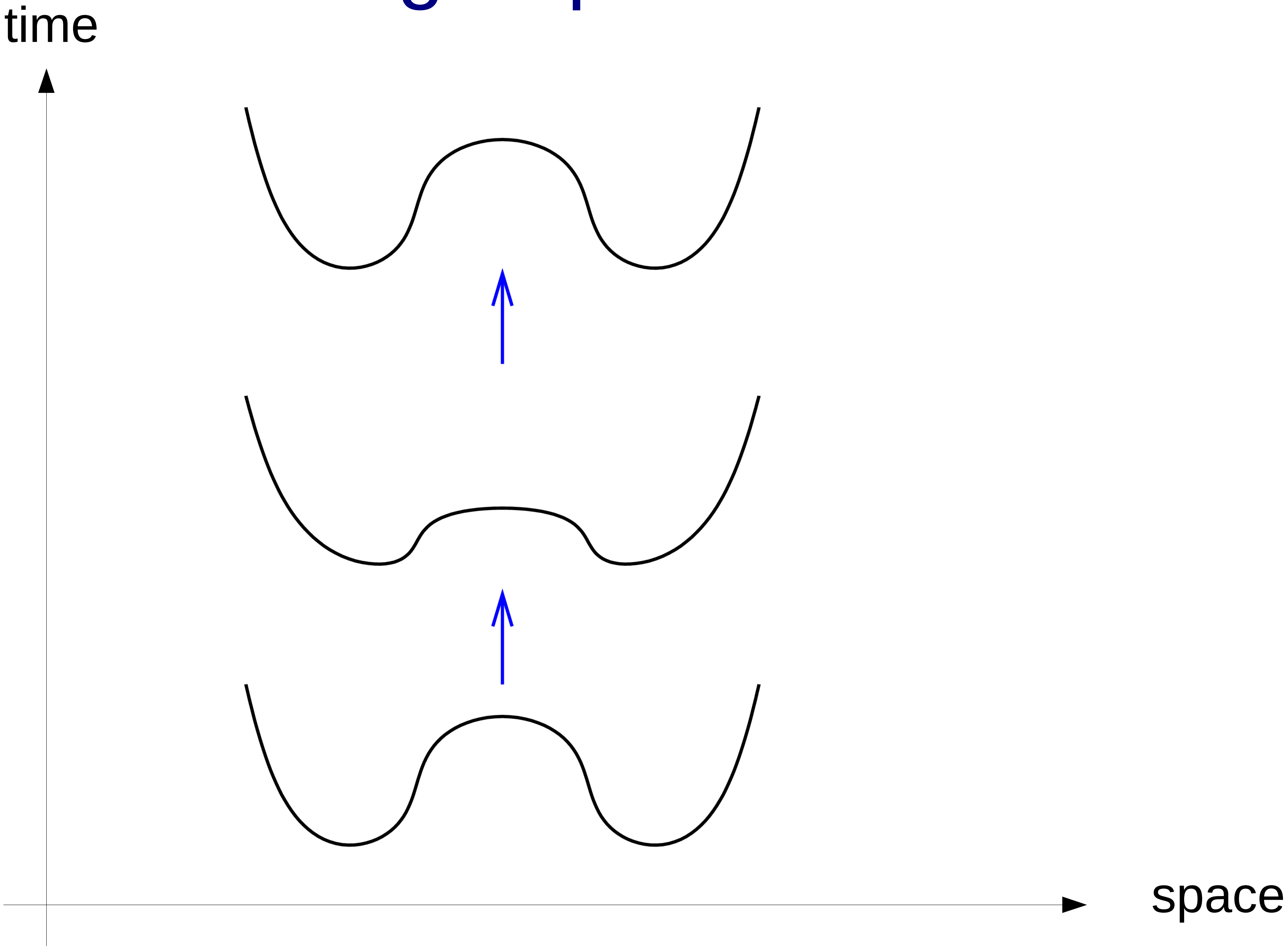
$|1\rangle$



Single-qubit Z-Rotation



Single-qubit X-Rotation

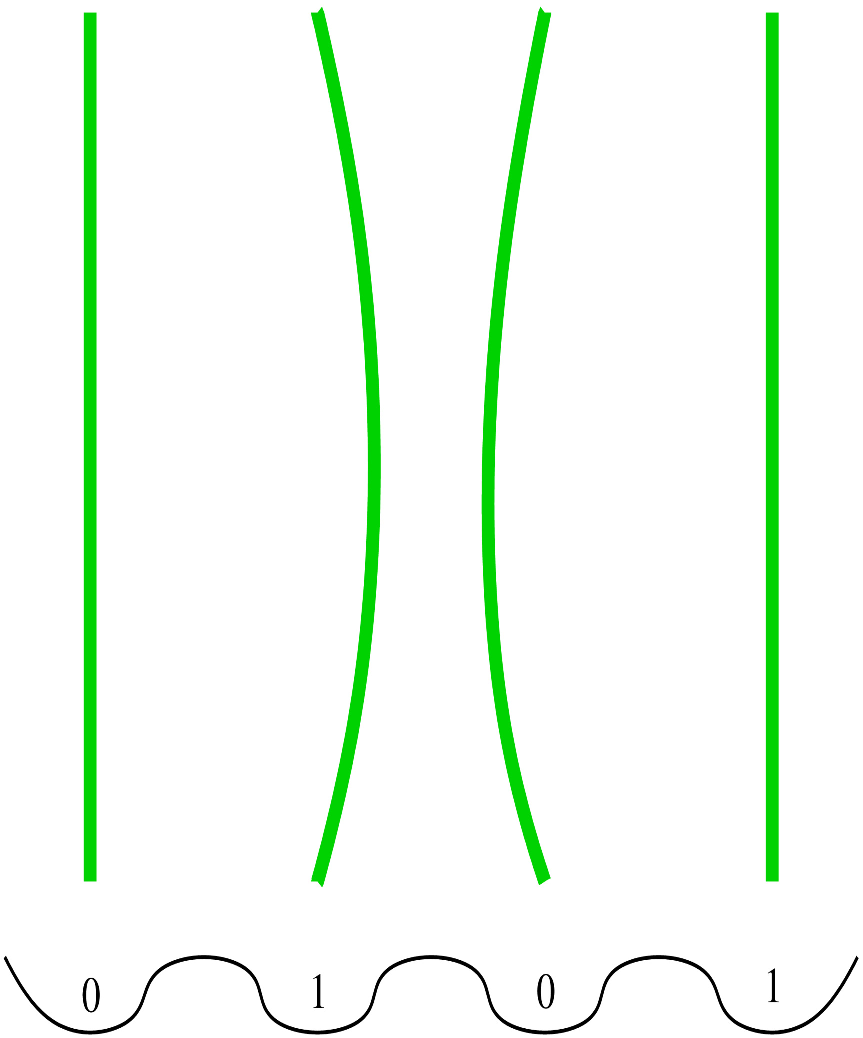


2-Qubit Entangling Gate

$$H_{\mathrm{NR}} = \sum_j \left(-\frac{p_j^2}{2m} + \frac{1}{m} J_2(x_j) \right) + \boxed{O(\lambda)}$$

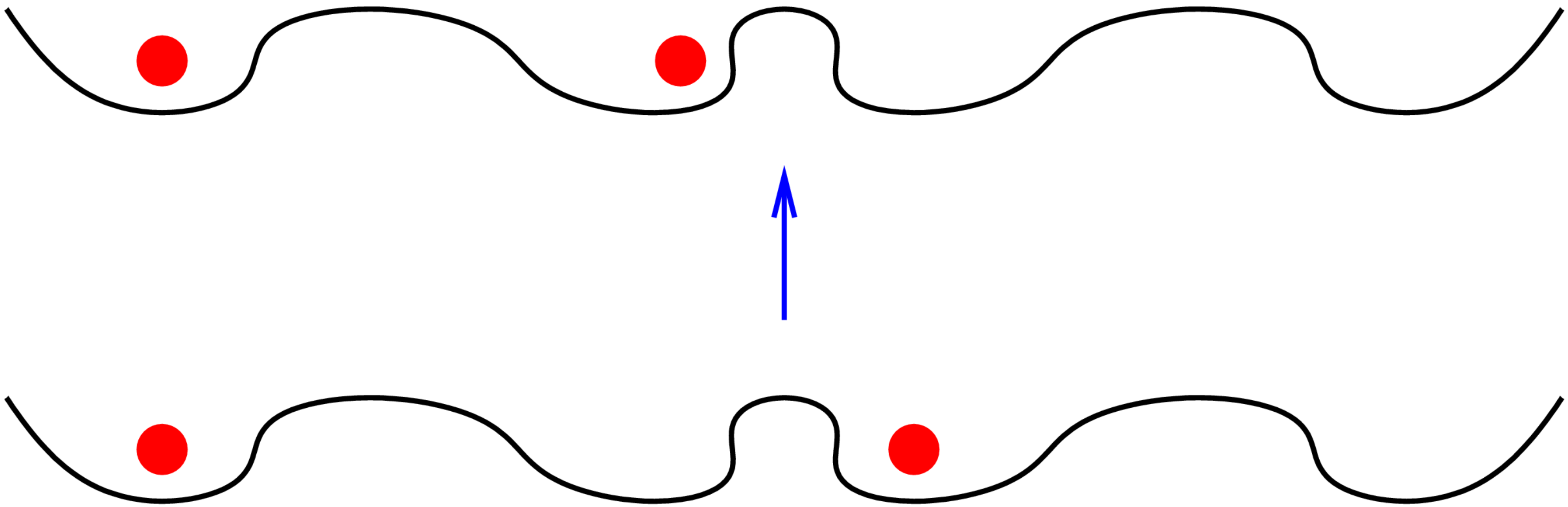
$$\frac{\lambda}{4m^2}\left(1+\frac{\lambda}{4\pi m^2}\right)\sum_{i<j}\delta(x_i-x_j)-\frac{\lambda^2}{32\pi m^3}\sum_{i<j}\int_0^1 dy\,\frac{e^{-mr_{ij}/\sqrt{y(1-y)}}}{\sqrt{y(1-y)}}$$

Our First Idea



$$\begin{aligned} |00\rangle &\mapsto & |00\rangle \\ |01\rangle &\mapsto & |01\rangle \\ |10\rangle &\mapsto & e^{-i\theta}|10\rangle \\ |11\rangle &\mapsto & |11\rangle \end{aligned}$$

Leakage



There is no regime in phi-fourth theory in which the tunneling between wells is negligible but the interparticle attraction is nonnegligible.

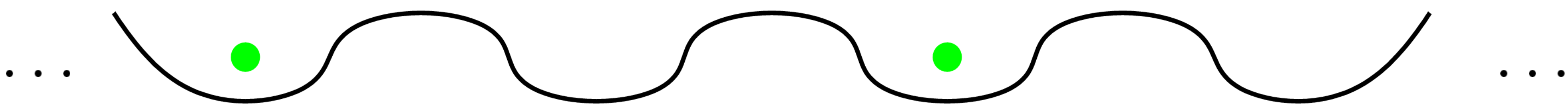
Solution

- Implement unitaries on full $\binom{4}{2}$-dimensional space.
- As a special case obtain:

$$\begin{bmatrix} \boxed{\mathrm{U}} & \\ & \boxed{?} \end{bmatrix}$$

Vacuum to Vacuum

- With the preceding method we can simulate an arbitrary quantum circuit.
- There's no error correction: each of the G gates has to be implemented with error ~1/G
- However, we started with a very special state:



- How do we start with vacuum?

Creating Particles: Rabi Oscillation

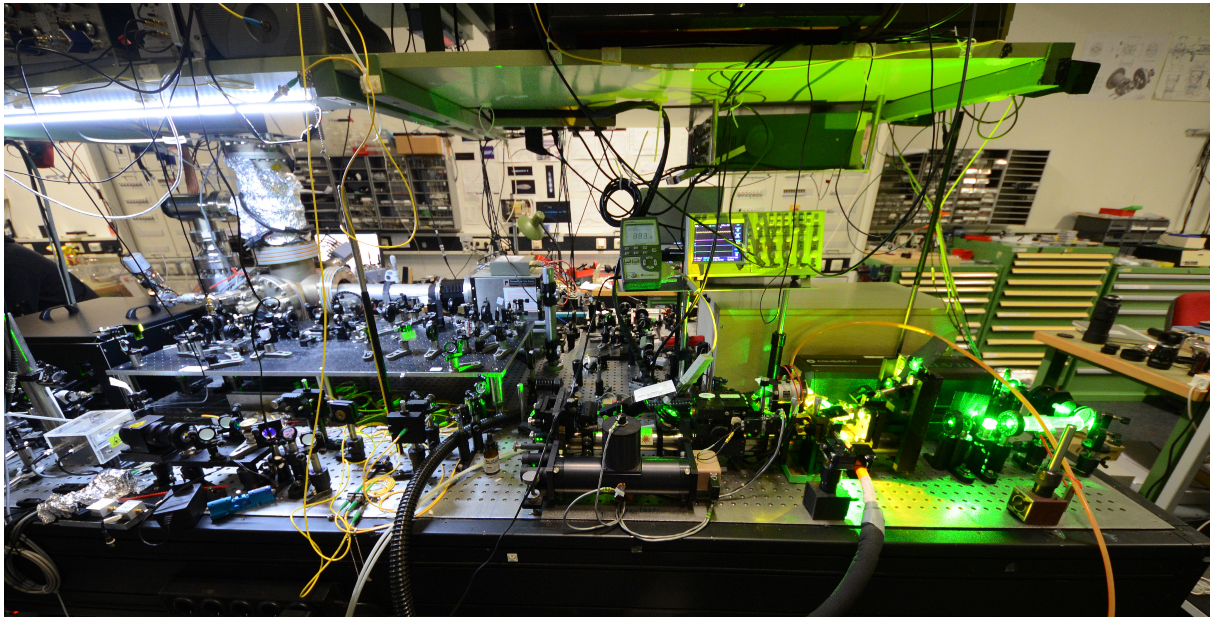
$$H = H_{\phi^4} + g\sin(\omega t)\int dx\ h(x)\phi(x)$$

A Difficulty

- We don't have error correction, so we need each of the n qubits to be prepared with error ~1/n
- In Rabi oscillation method this means:
  - We need $\omega = m \pm O(1/n)$
  - We need $\langle 1 | g \int dx \; h(x) \phi(x) | \mathrm{vac} \rangle = \frac{\pi}{2} \pm O(1/n)$
- This is not so easy!

Adiabatic Passage to the Rescue

- Sweep the driving frequency from below resonance to above resonance.
- In the limit that you sweep slowly and drive weakly, this errors provably converge to zero.
- Thanks, experimentalists!



[Photo: Udem group]

Conclusion

- Already a pretty simple problem in QFT is BQP-hard.
- One spatial dimension.
- Only massive Bosons.
- Weakly Coupled.
- Spacetime-varying source term.
- How about translationally invariant, or Lorentz-invariant systems?

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Thanks!