Quantum algorithms for simulating quantum field theories

John Preskill, Caltech

With: Stephen Jordan, Keith Lee, Hari Krovi

Quantum simulation of quantum field theories. Why?

QFT encompasses all fundamental interactions, possibly excluding gravity.

Can quantum computers efficiently simulate any process that occurs in Nature? (Quantum Church-Turing thesis.)

YES and NO are both exciting answers!

Event generators for QCD, etc.

Simulations of nuclear matter, etc.

Exploration of other strongly coupled theories.

Stepping stone to quantum gravity.

Characterizing computational complexity of quantum states.

New insights!

What problem does the algorithm solve?

Scattering problem: given initial (incoming) state, sample accurately from the distribution of final (outgoing) states.

Vacuum-to-vacuum probability in the presence of spacetime-dependent sources coupled to local observables.

Correlation functions, e.g., for insertions of unitary operators.

Correlation functions and bulk observables at nonzero temperature and chemical potential.

To probe, e.g., transport properties, formulate a simulation that models an actual experiment.

Digital vs. Analog quantum simulation

Analog is very active now, in both experiment and theory. Digital is more aspirational.

Platforms include: ultracold (neutral) atoms and molecules, trapped ions, superconducting circuits, etc.

There are ambitious proposals for simulating gauge field theories with existing experimental tools, e.g., using ultracold atoms.

High connectivity among qubits highly desirable (e.g., for probing scrambling of quantum information).

Limited by imperfect control. Does a noisy (analog) simulation perform a super-classical computational task?

This talk concerns (error corrected) digital quantum simulation.

Quantum field theory: some history

Around 1927: Dirac, Heisenberg, Jordan, Pauli, Born, …

Quantizing free electromagnetic field → photons. Reasonable results from first-order perturbation theory.

Infinities in higher-order perturbation theory. Is something wrong with quantum mechanics or quantum field theory?

Around 1948: Schwinger, Feynman, Tomonaga, Dyson, …

Think operationally! Express results in terms of experimentally measurable quantities (e.g., electron mass and charge). Infinities cancel.

Around 1956: Wightman, … Rigorous, but nonconstructive formulation of relativistic quantum field theory.

Around 1970: Wilson. "Continuum limit" of a lattice field theory as a second-order phase transition.

Why Ken Wilson is my hero

He answered the question: What is quantum field theory?

He understood the meaning of renormalization.

Formally, QFT has an infinite number of degrees of freedom per unit volume. (That sounds hard to simulate!)

But … the infrared physics does not depend sensitively on the ultraviolet physics (absorbed into small number of renormalized parameters). "Universality"

This makes physics possible (!), but makes exploration of UV physics difficult.

Wilson's insights flowed from thinking about how to simulate QFT on a digital computer.

Further insights from simulating QFT on a quantum computer?

Or … an answer to: What is string theory?

About Rigor

We can rigorously define relativistic quantum field theory (Wightman axioms). Can't yet do that for string theory.

Rigorous constructions are possible for "superrenormalizable" theories in D < 4 dimensions. (D = spacetime dimension.)

No fully rigorous construction of D = 4 asymptotically free QFT (like quantum chromodynamics).

Almost rigorous: 4D ϕ^4 theory has trivial (free) continuum limit. (But interesting to simulate for finite lattice spacing.

With few exceptions, D > 4 theories are free.

Our analysis of algorithms is precise where possible, nonrigorous where necessary.

Example: use of perturbation theory to estimate how error scales with lattice spacing.

Good News / Bad News

Quantum computers simulate quantum systems in real time, not imaginary time.

That's a shame, because imaginary time evolution (in some cases) is an efficient way to prepare ground states.

But it's okay, because Nature evolves in real time, too.

And simulation of real time evolution for highly entangled quantum many-body systems (including quantum field theories) is presumed to be hard classically.

We work with the Hamiltonian (not the action), so Lorentz covariance is not manifest. We have to pick an inertial frame, but can obtain frame-independent results (if we're careful).

Prototypical quantum simulation task

(1) State preparation. E.g., incoming scattering state.

(2) Hamiltonian evolution.

(3) Measure an observable. E.g., a simulated detector.

Goal: sample accurately from probability distribution of outcomes.

Determine how computational resources scale with: error, system size, particle number, total energy of process, mass gap, …

Resources include: number of qubits, number of gates, …

Hope for polynomial scaling! Or even better: polylog.

This is the quantum killer app (as Feynman knew). But what should we simulate, and what do we stand to learn?

Preparing the ground state of a local Hamiltonian

Can be NP-hard even for a classical spin glass.

And even harder (QMA-hard) for some quantum systems, even in 1D.

But if the state/process exists in Nature, we can hope to simulate it (QCT thesis).

Same goes for Gibbs states (finite temperature and chemical potential) and states far from equilibrium.

Where did the observed state of our universe come from? That's a question about cosmology …

Why not do what Andrew does?

Consider, e.g., a spin system on a lattice. Hamiltonian is a sum of k-local terms, each with O(1) operator norm.

Simulation cost for evolution operator U(t) is optimal in error (~ log(1/error), nearly linear in evolution time t.

But QFT has (formally) unbounded number of sites and unbounded operators at each site.

Our cost scales like poly(1/error). The obstacle is the lattice spacing, not the Trotter error.

Can we do better?

How to regulate?

Momentum space is natural for diagonalizing free field theory Hamiltonian, and formulating perturbation theory.

Renormalization group can also be formulated in momentum space.

But real space is better suited for simulation, because the Hamiltonian is spatially local.

We define fields on lattice sites, with lattice spacing a, a source of error. "Bare" parameters. Smaller lattice spacing means better accuracy, but more qubits to simulate a specified spatial volume.

Fields and their conjugate momenta are unbounded operators. We express them in terms of a bounded number of qubits, determined by energy of the simulated process.

Doing better: RG-improved lattice Hamiltonians? Tensor network constructions, e.g., c-MPS, c-MERA, wavelets?

What to simulate?

For example, a self-coupled scalar field in D=2, 3, 4.

$$H = \int d^{D-1}x \left(\frac{1}{2}\Pi^2 + \frac{1}{2}(\nabla\phi)^2 + \frac{1}{2}m_0^2\phi^2 + \frac{1}{4!}\lambda_0\phi^4 \right)$$

Without the ϕ^4 term, a Gaussian theory which is easy to simulate classically (noninteracting particles).

With this interaction term, particles can scatter. The dimensionless coupling parameter is λ/m^{4-D}. Classical simulations are hard when the coupling is strong.

Hardness persists even at weak coupling, if we want high precision, or if particles interact long enough to become highly entangled.

Summing perturbation theory is infeasible, and misses nonperturbative effects.

We assume theory has a mass gap.

A simulation protocol

Input: a list of incoming particle momenta.
Output: a list of (perhaps many) outgoing particle momenta.

Procedure:
(1) Prepare free field vacuum ($\lambda_0 = 0$).
(2) Prepare free field wavepackets (narrow in momentum).
(3) Adiabatically turn on the (bare) coupling.
(4) Evolve for time t with Hamiltonian H.
(5) Adiabatically turn off interaction.
(6) Measure field modes.

Assume no phase transition blocks adiabatic state preparation.

Alternative: create particles with spacetime dependent classical sources (better if there are bound states). Simulate detector POVM.

Lorentz invariance brutally broken in lattice theory, recovered by tuning bare H. (Ugh.) Also tune for achieve $ma \ll 1$.

Sources of error?

Nonzero lattice spacing a.

Finite spatial volume V.

Discretized fields and conjugate momenta.

Nonzero Trotter step size for simulation of time evolution.

Diabatic errors from turning coupling on and off.

Example: ϕ^4 theory in D=3 spacetime dimensions

Error ε scales with lattice spacing a as $\varepsilon = O(a^2)$.

Number of qubits Ω needed to simulate physical volume V is
$\Omega = V/a^2 = O(1/\varepsilon)$.

Gaussian state preparation (matrix arithmetic) uses $\Omega^{2.273}$ gates.
(though a customized algorithm exploiting translation invariance
does better).

Scaling with energy E: number of gates = $O(E^6)$.

Factor E from Trotter error, E^2 from lattice spacing $a \sim 1/E$, E^3 from
diabatic error.

Dominant diabatic error comes from splitting of $1 \rightarrow 3$ particles, for
which energy gap $\sim m^2/E$.

Thousands of logical qubits for $2 \rightarrow 4$ scattering with 1% error at $E/m = O(1)$. Yikes!

Challenges and Opportunities

Improving resource costs, greater rigor, phase transitions.

Simulations with near-term quantum devices.

Gauge fields, QCD, standard model.

Massless particles, chiral fermions, SUSY.

Conformal field theory, holography, chaos.

Alternative paradigms, e.g. conformal bootstrap.

Fresh approaches to fault-tolerant quantum simulation.