TEL AVIV UNIVERSITY אוניברסיטת תל אביב

SIMULATING LATTICE GAUGE THEORIES WITH COLD ATOMS

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In collaboration with
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MPQ

Workshop on Computational Complexity and High Energy Physics
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University of Maryland

OUTLINE

QUANTUM SIMULATION

COLD ATOMS IN OPTICAL LATTICES

HAMILTONIAN LATTICE GAUGE THEORY (LGT)

ANALOG SIMULATION: LGT

- REQUIREMENTS
- EXACT AND EFFECTIVE LOCAL GAUGE INVARIANCE
- LINKS AND PLAQUETTES – Examples: cQED, SU(2)

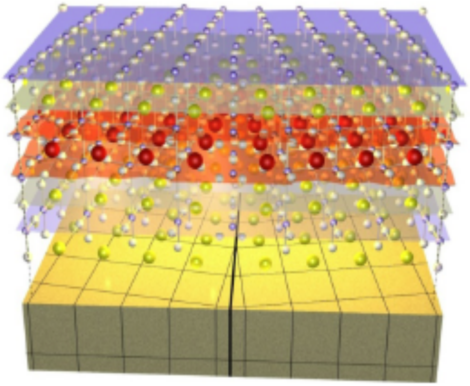
DIGITAL SIMULATION

CURRENT EXPERIMENTS

OUTLOOK.

QUANTUM SIMULATION
ANALOG

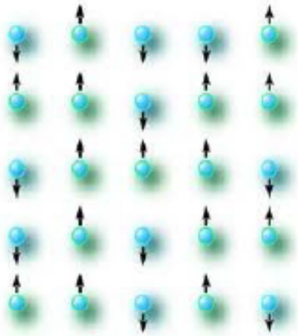
PHYSICAL SYSTEM



(Phenomenological) Hamiltonian

$H = \ldots$

QUANTUM SIMULATOR



Physical Hamiltonian

$H = \ldots$

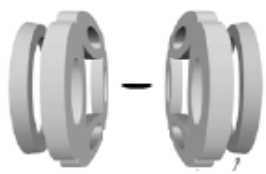
COLD ATOMS

- Control: External fields

Trapping:

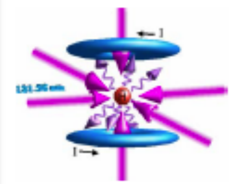
lasers

Magnetic fields



Cooling:

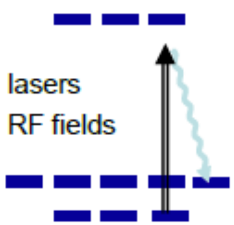
lasers



evaporation

Internal manipulation

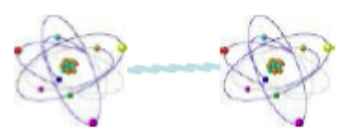
lasers
RF fields



purification
coherence
detection

Interactions

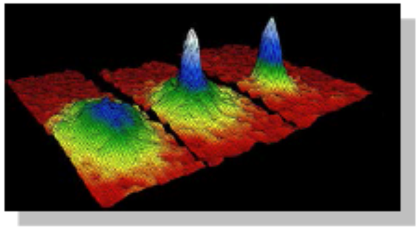
tune scattering length

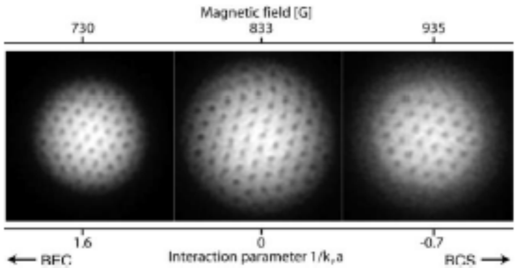


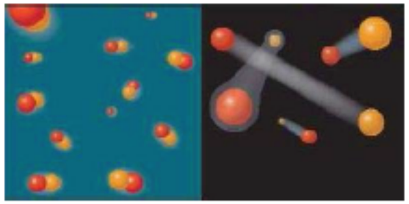
COLD ATOMS

- Many-body phenomena
  - Degeneracy: bosons and fermions (BE/FD statistics)
  - Coherence: interference, atom lasers, four-wave mixing, …
  - Superfluidity: vortices
  - Disorder: Anderson localization
  - Fermions: BCS-BEC

+ many other phenomena







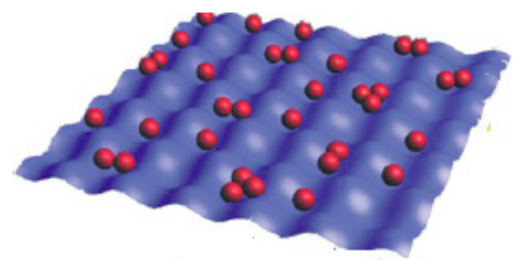
COLD ATOMS
OPTICAL LATTICES

- Laser standing waves: dipole-trapping

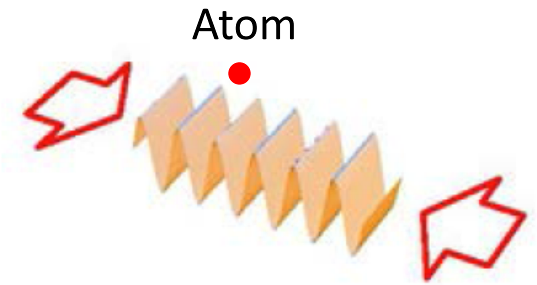
VOLUME 81, NUMBER 15 PHYSICAL REVIEW LETTERS 12 OCTOBER 1998

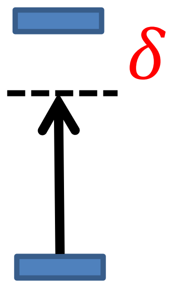
Cold Bosonic Atoms in Optical Lattices

D. Jaksch,[1,2] C. Bruder,[1,3] J. I. Cirac,[1,2] C. W. Gardiner,[1,4] and P. Zoller[1,2]



COLD ATOMS
OPTICAL LATTICES

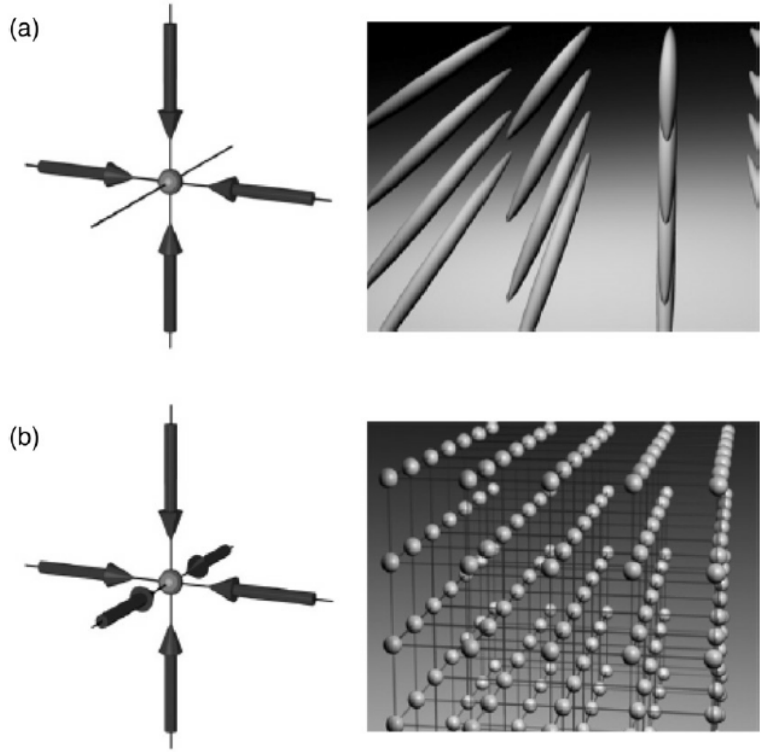




In the presence $\boldsymbol{E}(r,t)$ the atoms has a time dependent dipole moment $d(t) = \alpha(\omega)\, \boldsymbol{E}(r,t)$ of some non resonant excited states.
Stark effect:

$$\mathrm{V}(\mathrm{r}) \equiv \Delta\mathrm{E}(\mathrm{r}) = \alpha(\omega)\langle\, \boldsymbol{E}(r,t)\, \boldsymbol{E}(r,t)\rangle/\delta$$

COLD ATOMS
OPTICAL LATTICES



(a) 2d array of effective 1d traps
(b) 3d square lattice

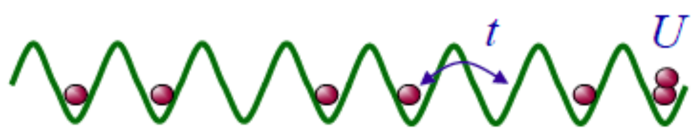
M. Lewenstein et. al, Advances in Physics, 2010.

COLD ATOMS

OPTICAL LATTICES

- Laser standing waves: dipole-trapping

$$H = \int \Psi_\sigma^\dagger \left(-\nabla^2 + V(r)\right)\Psi_\sigma + u_{\sigma_i} \int \Psi_{\sigma_1}^\dagger \Psi_{\sigma_2}^\dagger \Psi_{\sigma_3} \Psi_{\sigma_4}$$



Lattice theory: Bose/Fermi-Hubbard model

$$H = -t\sum_n \left(a_n^\dagger a_{n+1} + h.c\right) + U\sum_n a_n^{\dagger 2} a_n^2$$

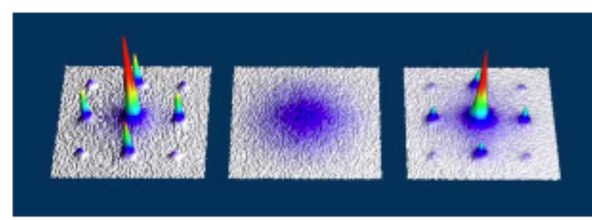
articles

Quantum phase transition from a superfluid to a Mott insulator in a gas of ultracold atoms

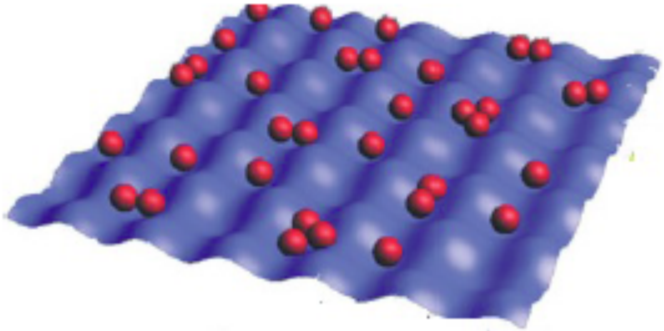
Markus Greiner*, Olaf Mandel*, Tilman Esslinger†, Theodor W. Hänsch* & Immanuel Bloch*

* Sektion Physik, Ludwig-Maximilians-Universität, Schellingstrasse 4/III, D-80799 Munich, Germany, and Max-Planck-Institut für Quantenoptik, D-85748 Garching, Germany

† Quantenelektronik, ETH Zürich, 8093 Zurich, Switzerland



COLD ATOMS
QUANTUM SIMULATIONS

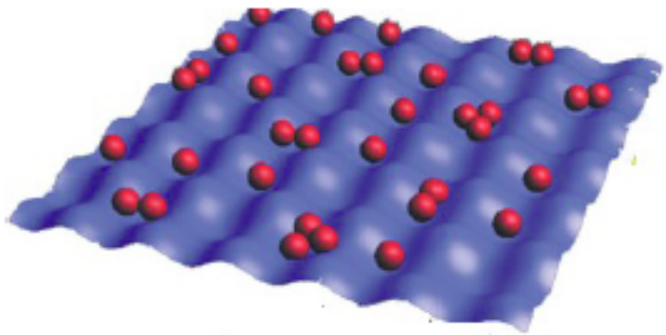


- Bosons/Fermions: $H = -\sum_{\substack{<n,m> \\ \sigma,\sigma'}} \left(t_{\sigma,\sigma'} a^{\dagger}_{n,\sigma} a_{m,\sigma'} + h.c \right) + \sum_{\substack{n \\ \sigma,\sigma'}} U_{\sigma,\sigma'} a^{\dagger}_{n,\sigma} a^{\dagger}_{n,\sigma'} a_{n,\sigma'} a_{n,\sigma}$

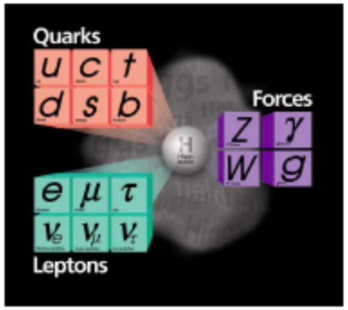
- Spins: $H = -\sum_{\substack{<n,m> \\ \sigma,\sigma'}} \left(J_x S^x_n S^x_m + J_y S^y_n S^y_m + J_z S^z_n S^z_m \right) + \sum_{\substack{n \\ \sigma,\sigma'}} B_n S^z_n$

➡ CONDENSED MATTER PHYSICS

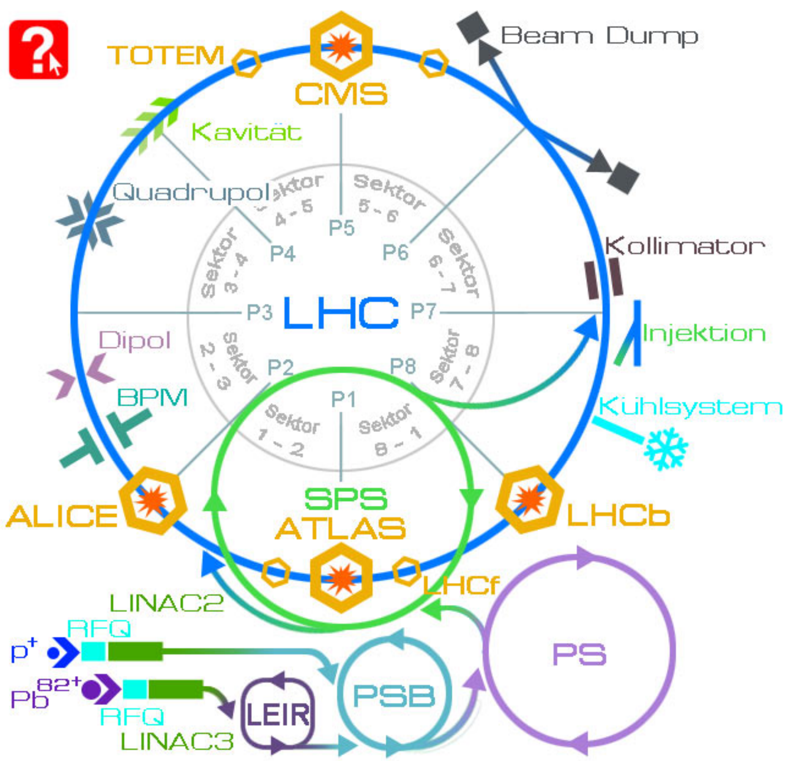
COLD ATOMS
QUANTUM SIMULATIONS

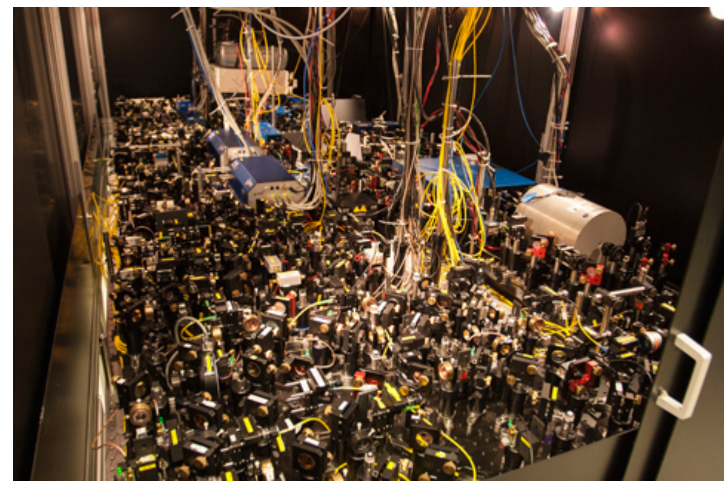


HIGH ENERGY PHYSICS?



24 < ORDERS OF MAGNITUDE!!

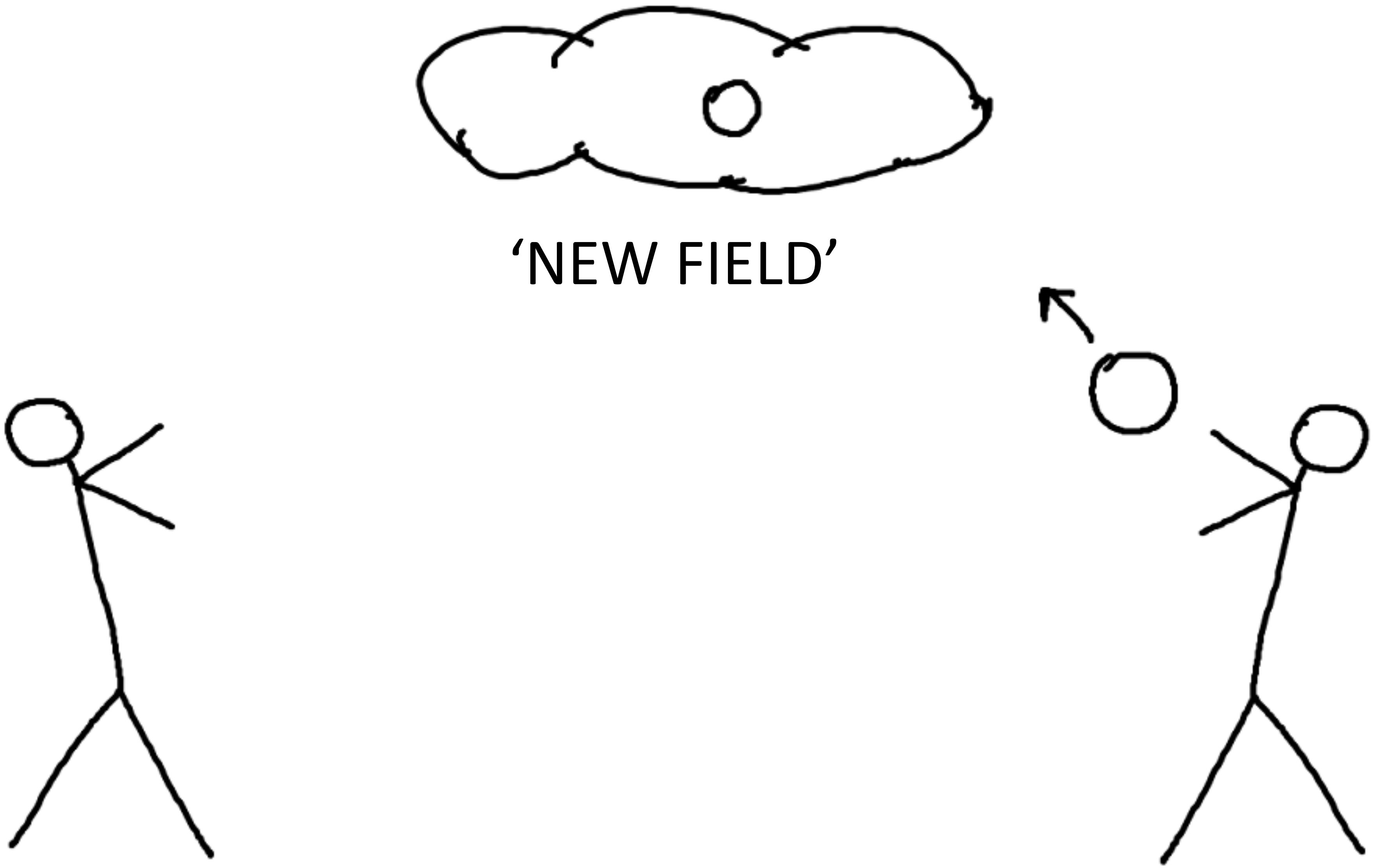




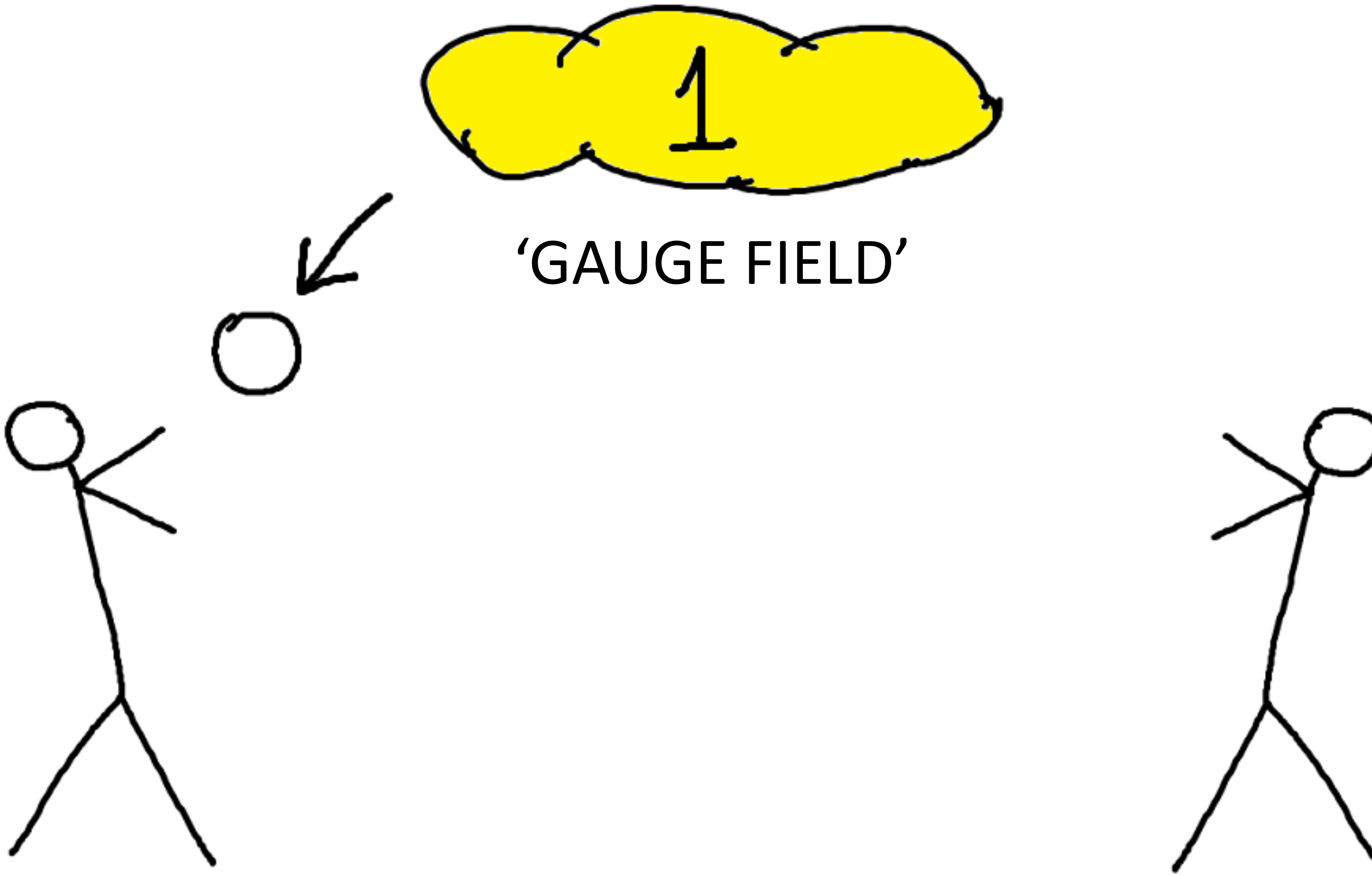
LONG RANGE FORCES?

LONG RANGE FORCES?

REQUIRE FORCE CARRIER



REQUIRE FORCE CARRIER



THE STANDARD MODEL

- Matter:= fermions

(Quarks and Leptons w.

mass, spin 1/2, flavor, charge)

- Interactions mediators := YM gauge fields

(spin 1 bosons) .

Electromagnetic: massless chargeless photon, (1), U(1)

Weak interaction : massive, charged Z, W's , (3), SU(2)

Strong interaction : massless Gluons , (8), SU(3)

GAUGE FIELDS

Abelian Fields **Maxwell**	**Non-Abelian fields** **Yang-Mills**
Massless	Massless
Long-range forces	**<u>Confinement</u>**
Chargeless	**<u>Carry charge</u>**
Linear dynamics	**<u>Self interacting & NL</u>**

QED

$$\alpha_{QED} \ll 1, \quad V_{QED}(r) \propto \frac{1}{r}$$

We (ordinarily) don’t need QFT quantum field theory to understand the structure of atoms:

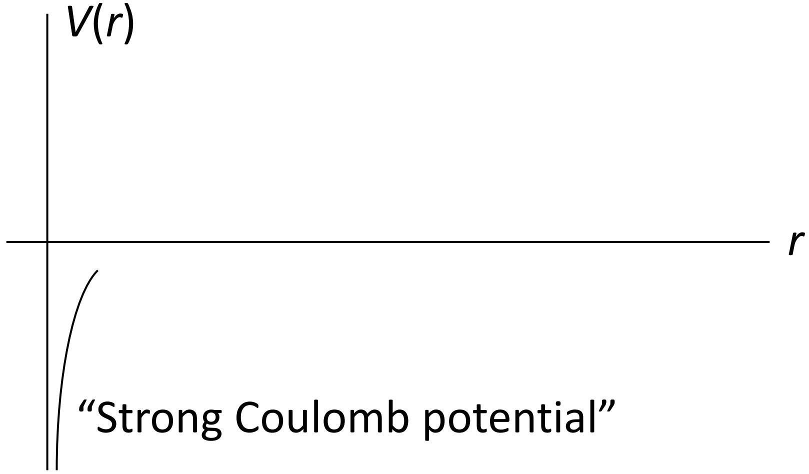
$$m_e c^2 \gg E_{Rydberg} \simeq \alpha_{QED}^2 \, m_e c^2$$

But also higher energies effects are well described using perturbation theory - (Feynman diagrams) works well.

QCD: AT HIGH ENERGY
ASYMPTOTIC FREEDOM

- Quantum Chromodynamics asymptotic freedom:
  at high energies, coupling constant 'goes' to zero.

- The nucleus, are seen as built of 'free' point-like particles= quarks.



QCD: AT LOW ENERGIES
ASYMPTOTIC FREEDOM

$$\alpha_{QCD} > 1\,, V_{QCD}(r) \propto r$$

Non-perturbative confinement effect

No free quarks! they construct Hadrons:

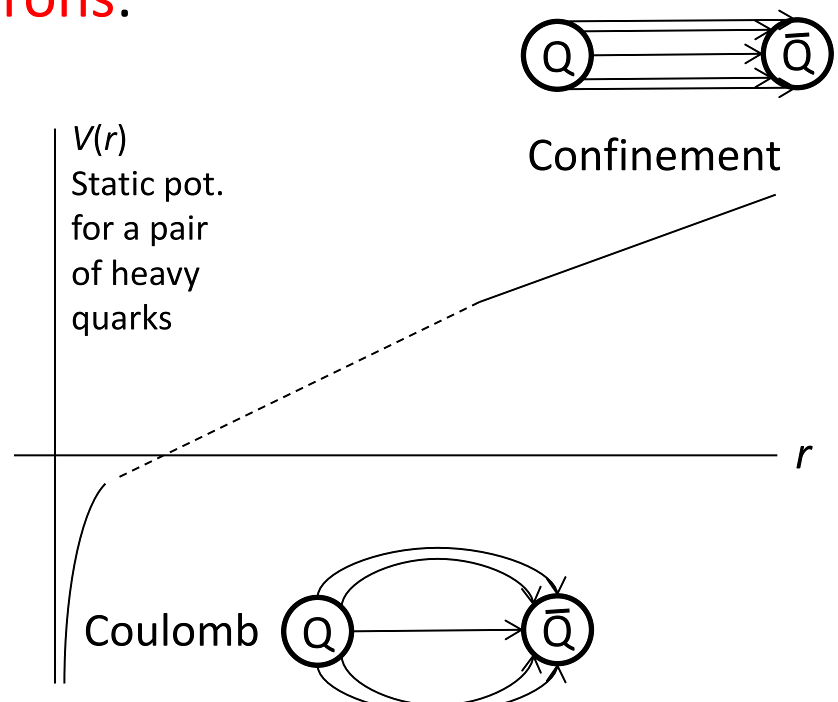
Mesons (two quarks),

Baryons (three quarks),

...

Color Electric flux-tubes:

"a non-abelian Meissner effect".



(some) OPEN PROBLEMS

–Mass gap of Yang-Mills (pure gauge) theories.

–Phases of non-Abelian theories with fermionic matter

–Color superconductivity?

– Quark-gluon Plasma.

– Confinement/deconfinement of dynamical charges

–High-*T*c superconductivity ?

LATTICE GAUGE THEORIES

We are all Wilsonians now

Posted on June 18, 2013 by preskill

Ken Wilson passed away on June 15 at age 77. He changed how we think about physics.

Renormalization theory, first formulated systematically by Freeman Dyson in 1949, cured the flaws of quantum electrodynamics and turned it into a precise computational tool. But the subject seemed magical and mysterious. Many physicists, Dirac prominently among them, questioned whether renormalization rests on a sound foundation.

Wilson changed that.

The renormalization group concept arose in an extraordinary paper by

Wilson also formulated the strong-coupling expansion of lattice gauge theory, and soon after pioneered the Euclidean Monte Carlo method for computing the quantitative non-perturbative predictions of quantum chromodynamics, which remains today an extremely active and successful program. But of the papers by Wilson I read while in graduate school, the most exciting by far was this one about the renormalization group. Toward the end of the paper Wilson discussed how to formulate the notion of the "continuum limit" of a field theory with a

LATTICE GAUGE THEORIES

HAMILTONIAN FORMULATION

PHYSICAL REVIEW D VOLUME 11, NUMBER 2 15 JANUARY 1975

Hamiltonian formulation of Wilson's lattice gauge theories

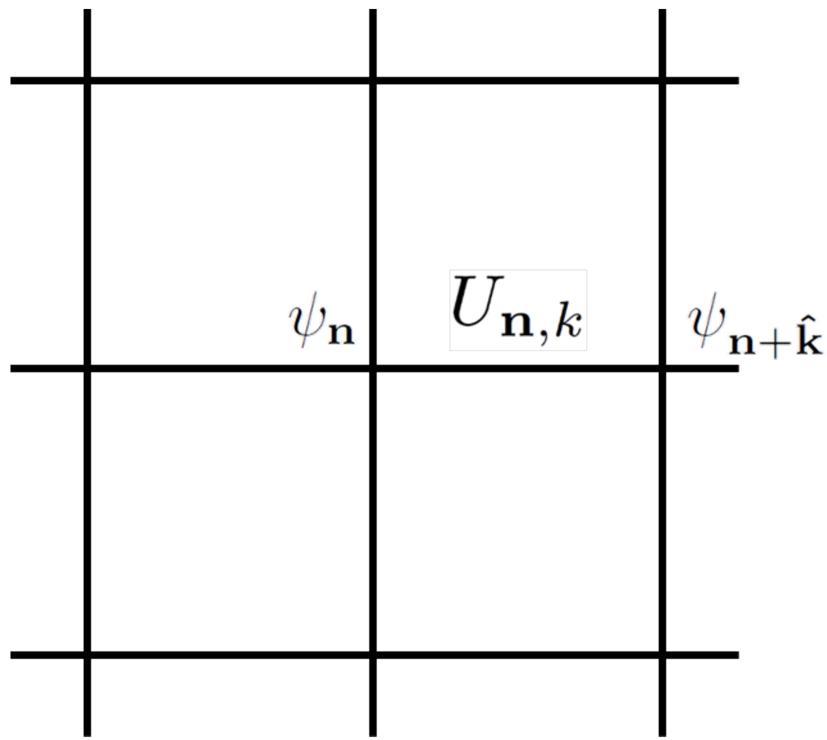
John Kogut*
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14853

Leonard Susskind†
Belfer Graduate School of Science, Yeshiva University, New York, New York
and Tel Aviv University, Ramat Aviv, Israel
and Laboratory of Nuclear Studies, Cornell University, Ithaca, New York

(Received 9 July 1974)

Wilson's lattice gauge model is presented as a canonical Hamiltonian theory. The structure of the model is reduced to the interactions of an infinite collection of coupled rigid rotators. The gauge-invariant configuration space consists of a collection of strings with quarks at their ends. The strings are lines of non-Abelian electric flux. In the strong-coupling limit the dynamics is best described in terms of these strings. Quark confinement is a result of the inability to break a string without producing a pair.

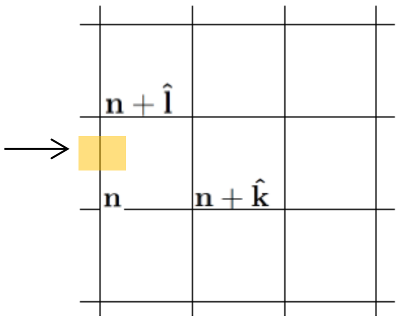
LATTICE GAUGE THEORY



LATTICE GAUGE THEORIES
HAMILTONIAN FORMULATION

Gauge group elements:

U^r is an element of the gauge group (in the representation r), on each link



Left and right generators:

$$[L_a, U^r] = T_a^r U^r \quad ; \quad [R_a, U^r] = U^r T_a^r$$

$$[L_a, L_b] = -i f_{abc} L_c \quad ; \quad [R_a, R_b] = i f_{abc} R_c \quad ; \quad [L_a, R_b] = 0$$

$$\sum_a L_a L_a = \sum_a R_a R_a \equiv \sum_a E_a E_a$$

Gauge transformation:

$$U^r_{\mathbf{n},k} \rightarrow V^r_{\mathbf{n}} U^r_{\mathbf{n},k} V^{\dagger r}_{\mathbf{n}+\hat{\mathbf{k}}}$$

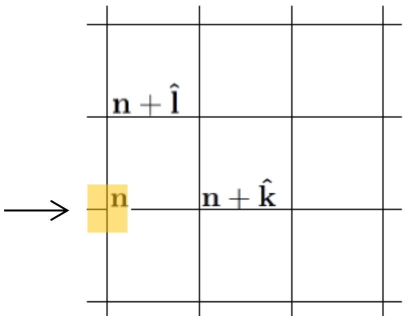
Generators:

$$(G_{\mathbf{n}})_a = \mathrm{div}_{\mathbf{n}} E_a = \sum_k \left((L_{\mathbf{n},k})_a - \left(R_{\mathbf{n}-\hat{\mathbf{k}},k} \right)_a \right)$$

LATTICE GAUGE THEORIES
HAMILTONIAN FORMULATION

Matter:

$$\psi_{\mathbf{n}} = (\psi_{\mathbf{n},a}) = \begin{pmatrix} \psi_{\mathbf{n},1} \\ \psi_{\mathbf{n},2} \\ \cdots \end{pmatrix}$$



Gauge transformation:

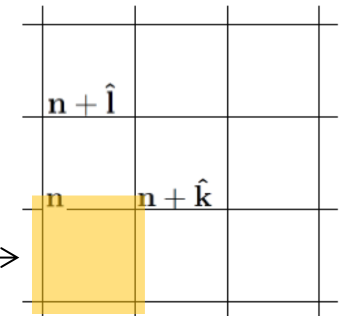
$$\psi_{\mathbf{n}} \to V^r_{\mathbf{n}} \psi_{\mathbf{n}}$$

LATTICE GAUGE THEORIES
HAMILTONIAN FORMULATION

Gauge field dynamics (Kogut-Susskind Hamiltonian):

$$H_E = \frac{g^2}{2}\sum_{\mathbf{n},k,a} \left(E_{\mathbf{n},k}\right)_a \left(E_{\mathbf{n},k}\right)_a$$

$$H_B = -\frac{1}{g^2}\sum_{\text{plaquettes}} \left(\mathrm{Tr}\left(U_1 U_2 U_3^\dagger U_4^\dagger\right) + h.c.\right)$$

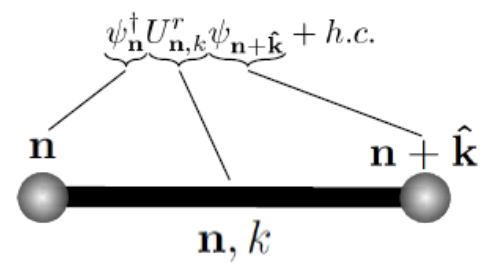


Strong coupling limit: $g >> 1$
Weak coupling limit: $g << 1$

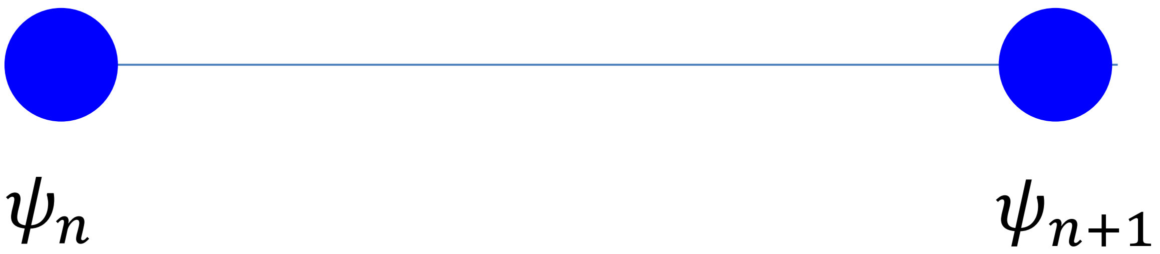
Matter dynamics:

$$H_M = \sum_{\mathbf{n}} M_{\mathbf{n}} \psi_{\mathbf{n}}^\dagger \psi_{\mathbf{n}}$$

$$H_{int} = \epsilon \sum_{\mathbf{n},k} \left(\psi_{\mathbf{n}}^\dagger U_{\mathbf{n},k}^r \psi_{\mathbf{n}+\hat{\mathbf{k}}} + h.c.\right)$$



TOY EXAMPLE: U(1)



$$H = \sum_n M_n \psi_n^\dagger \psi_n + \epsilon \sum_n \left(\psi_n^\dagger \psi_{n+1} + h.c. \right)$$

TOY EXAMPLE: U(1)

$$H = \sum_n M_n \psi_n^\dagger \psi_n + \epsilon \sum_n \left(\psi_n^\dagger \psi_{n+1} + h.c. \right)$$

H is invariant under global transformations:

$$\psi_n \longrightarrow e^{-i\Lambda} \psi_n \quad ; \quad \psi_n^\dagger \longrightarrow \psi_n^\dagger e^{i\Lambda}$$

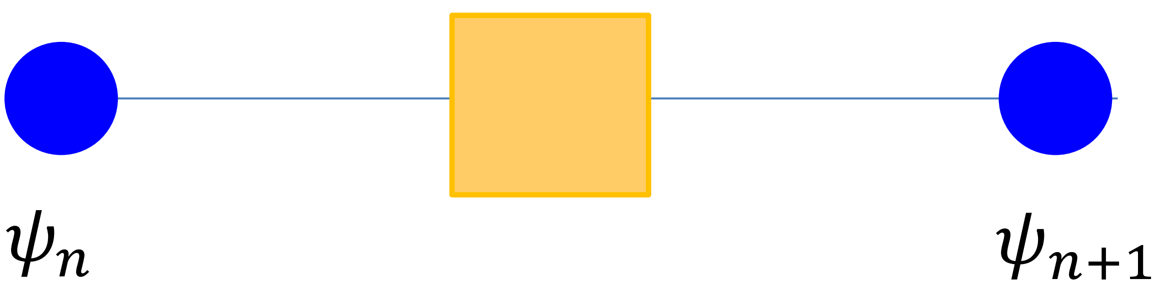
TOY EXAMPLE: U(1)

Promote the transformation to be local:

$$\psi_n \longrightarrow e^{-i\Lambda_n}\psi_n \quad ; \quad \psi_n^\dagger \longrightarrow \psi_n^\dagger e^{i\Lambda_n}$$

Add a **new field** on the links:

$$U_n = e^{i\phi_n}$$



TOY EXAMPLE: U(1)

$$H = \sum_n M_n \psi_n^\dagger \psi_n + \epsilon \sum_n \left(\psi_n^\dagger U_n \psi_{n+1} + h.c. \right)$$

Invariance under a **local** gauge transformations:

$$\psi_n \longrightarrow e^{-i\Lambda_n} \psi_n \quad ; \quad \psi_n^\dagger \longrightarrow \psi_n^\dagger e^{i\Lambda_n}$$

$$\phi_n \longrightarrow \phi_n + \Lambda_{n+1} - \Lambda_n$$

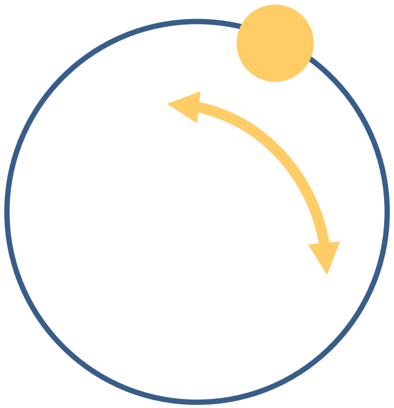
TOY EXAMPLE: U(1)

Gauge field dynamics:

$$H_E = \frac{g^2}{2}\sum_n L_n^2$$

$$L\left|m\right\rangle = m\left|m\right\rangle$$

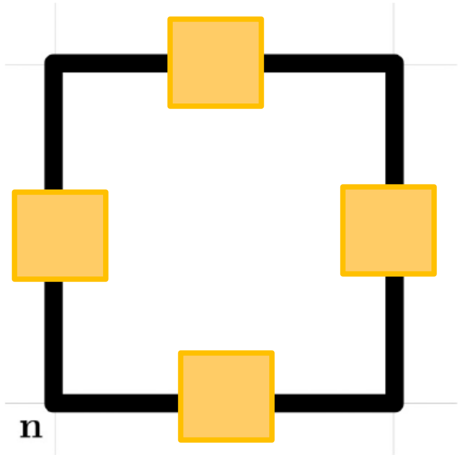
$$u_m\left(\phi\right) = \left\langle\phi|m\right\rangle = \frac{1}{\sqrt{2\pi}}e^{im\phi}$$



KS: “U(1) Rigid rotatator”

TOY EXAMPLE: U(1)

Gauge field dynamics: PLAQUETTES



$$H_B = -\frac{1}{2g^2}\sum_{plaquettes} U_1 U_2 U_3^\dagger U_4^\dagger + h.c. =$$

$$-\frac{1}{g^2}\sum_{plaquettes} \cos\left(\phi_1 + \phi_2 - \phi_3 - \phi_4\right)$$

In the continuum limit, this REDUCES to $(\nabla\times\boldsymbol{A})^2$: the magnetic energy density.

COMPACT QED (cQED)

$$U_{\mathbf{n},k} = e^{i\phi_{\mathbf{n},k}}$$

$$[E_{\mathbf{n},k}, \phi_{\mathbf{m},l}] = -i\delta_{\mathbf{nm}}\delta_{kl}$$

Electric energy + Magnetic energy + Gauge-Matter interaction

$$\frac{g^2}{2}\sum_{\mathbf{n},k} E^2_{\mathbf{n},k} - \frac{1}{g^2}\sum_{\mathbf{n}} \cos\left(\phi_{\mathbf{n},1} + \phi_{\mathbf{n}+\hat{\mathbf{1}},2} - \phi_{\mathbf{n}+\hat{\mathbf{2}},1} - \phi_{\mathbf{n},2}\right)$$

$$+ \epsilon \sum_{\mathbf{n},k} \left(\psi^\dagger_{\mathbf{n}} e^{i\phi_{\mathbf{n},k}} \psi_{\mathbf{n}+\hat{\mathbf{k}}} + \psi^\dagger_{\mathbf{n}+\hat{\mathbf{k}}} e^{-i\phi_{\mathbf{n},k}} \psi_{\mathbf{n}}\right)$$

$$+ M \sum_n (-1)^n \psi^\dagger_n \psi_n$$

QUANTUM SIMULATIONS

Simulating Physics with Computers

Richard P. Feynman

Department of Physics, California Institute of Technology, Pasadena, California 91107

Received May 7, 1981

1. INTRODUCTION

On the program it says this is a keynote speech—and I don't know what a keynote speech is. I do not intend in any way to suggest what should be in this meeting as a keynote of the subjects or anything like that. I have

be understood very well in analyzing the situation. And I'm not happy with all the analyses that go with just the classical theory, because nature isn't classical, dammit, and if you want to make a simulation of nature, you'd better make it quantum mechanical, and by golly it's a wonderful problem, because it doesn't look so easy. Thank you.

International Journal of Theoretical Physics, Vol. 21, Nos. 6/7, 1982

REQUIREMENTS: HEP models

- Fields

    Fermion Matter fields

    Bosonic gauge fields

- Local gauge invariance

    Exact, or low energy, effective

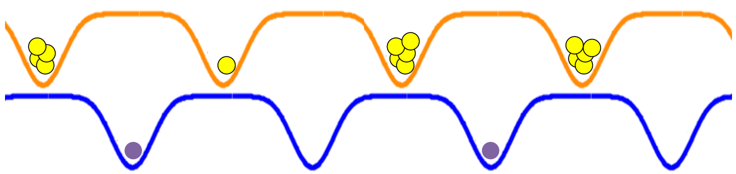
- Relativistic invariance

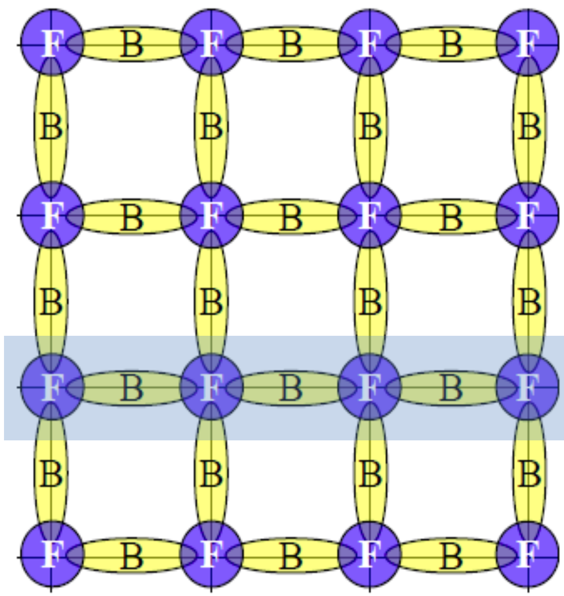
    Causal structure, in the continuum limit

QUANTUM SIMULATION
COLD ATOMS

- Fermion matter fields
- Bosonic gauge fields

Superlattices:



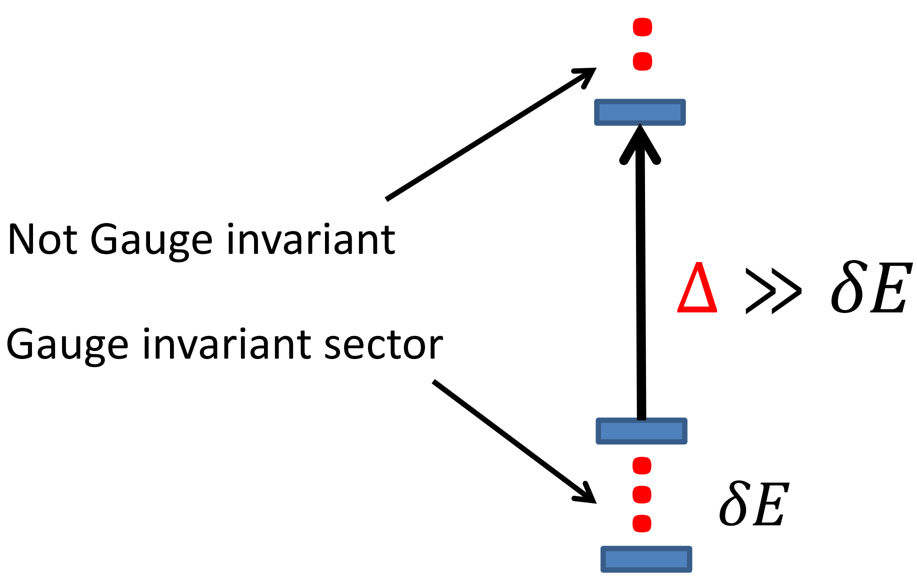


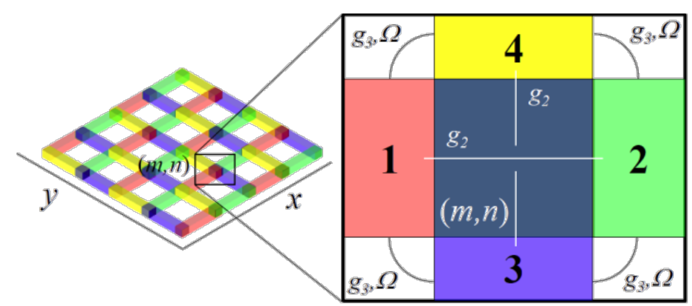
$$\psi_{\mathbf{n}} = (\psi_{\mathbf{n},a}) = \begin{pmatrix} \psi_{\mathbf{n},1} \\ \psi_{\mathbf{n},2} \\ \dots \end{pmatrix} \longrightarrow$$ Atom internal levels

1) EFFECTIVE GAUGE INVARIANCE

Gauss's law is added as a constraint. Leaving the gauge invariant sector of Hilbert space costs too much Energy.

Low energy sector with a effective gauge invariant Hamiltonian.





E. Zohar, B. Reznik, Phys. Rev. Lett. 107, 275301 (2011)

2) EXACT GAUGE INVARIANCE

- Atomic Symmetries $\leftrightarrow$ Local Gauge Invariance

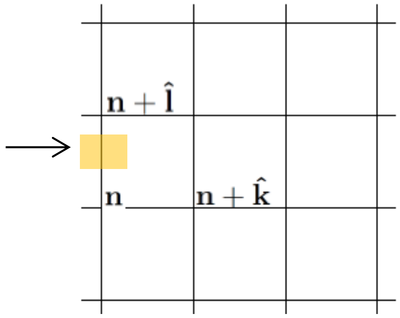
ABELIAN CASE:

E. Zohar, J. I. Cirac, B. Reznik, Phys. Rev. A **88** 023617 (2013)

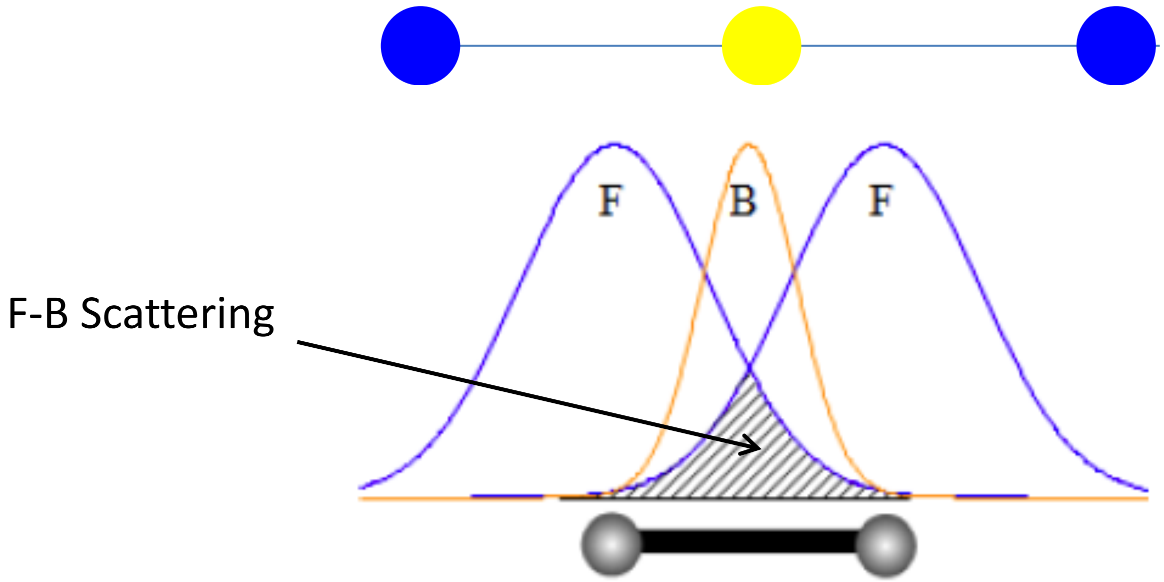
NON-ABELIAN CASE:

E. Zohar, J. I. Cirac, B. Reznik, Rep. Prog. Phys. **79**, 014401 (2016)

LINKS



cQED LINK



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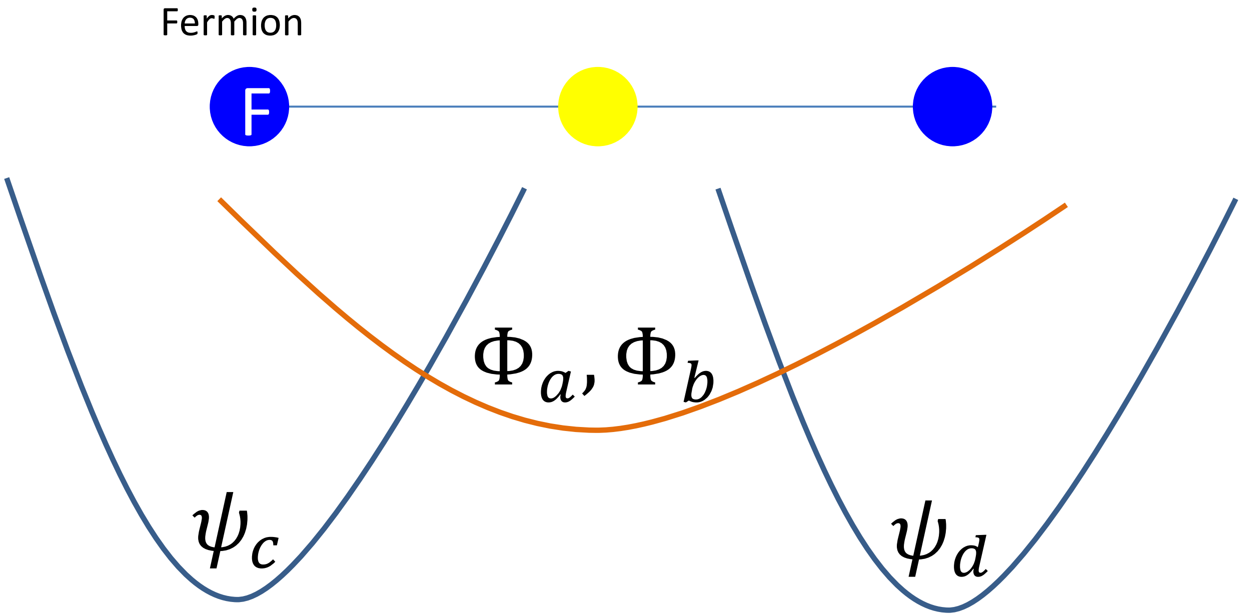

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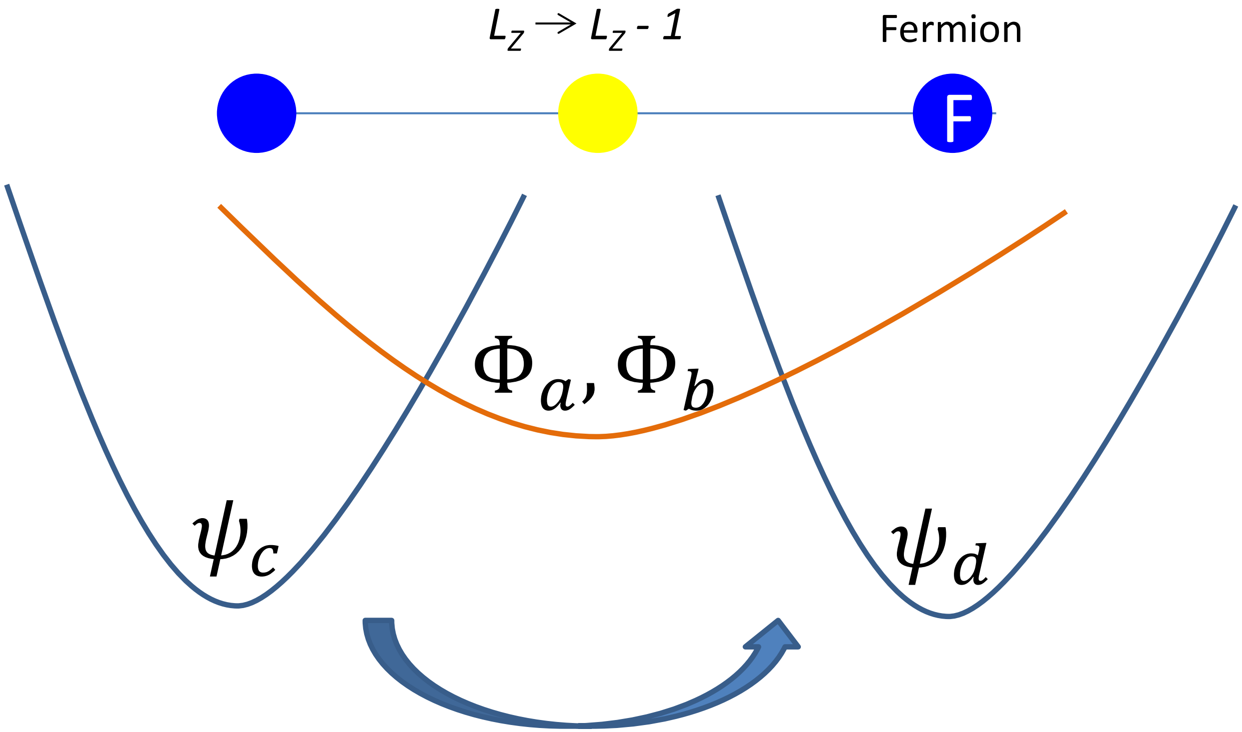

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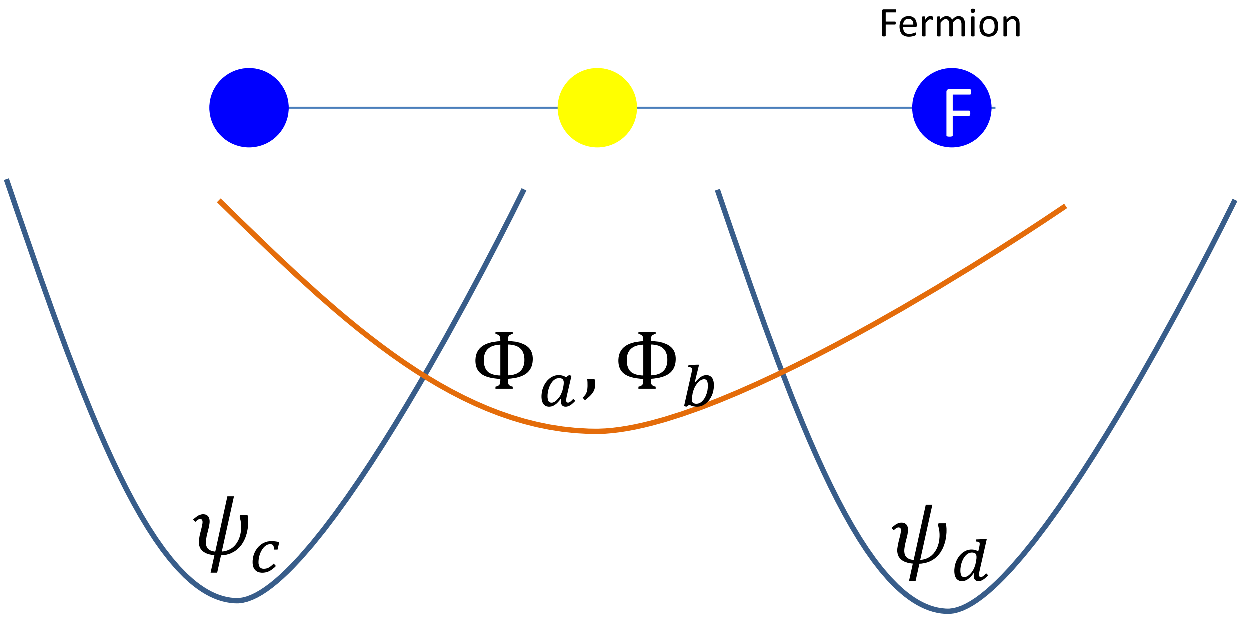

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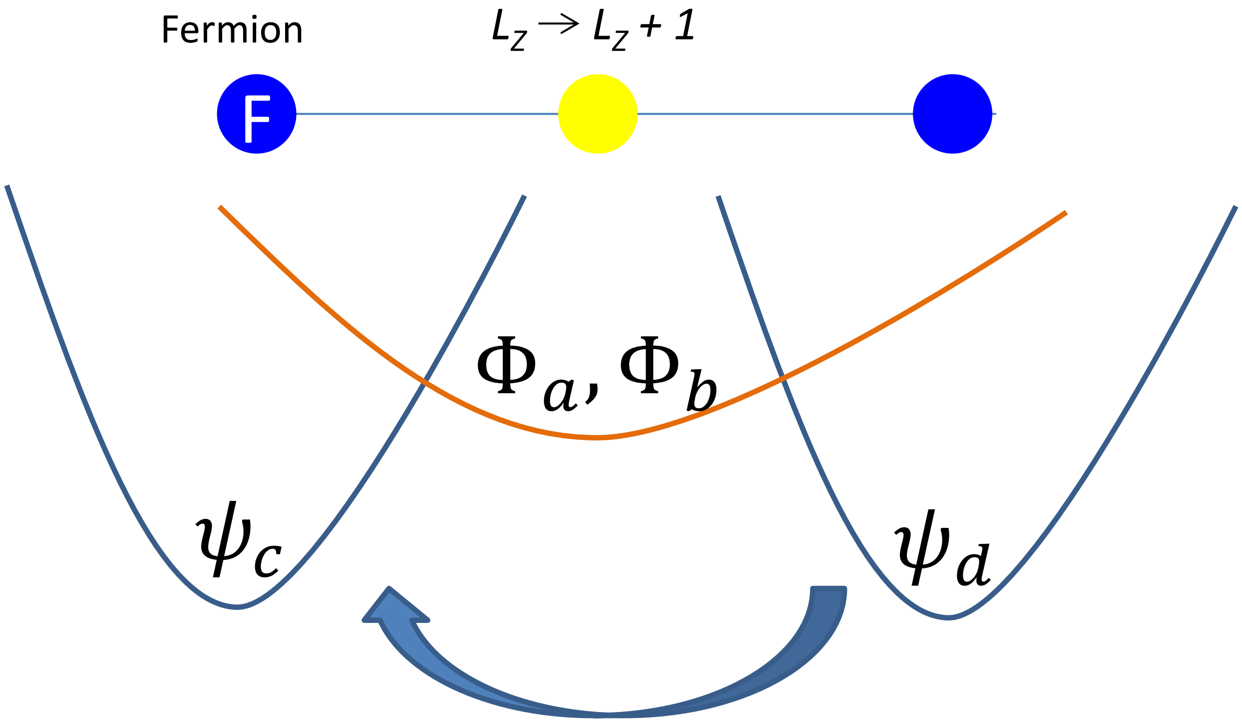

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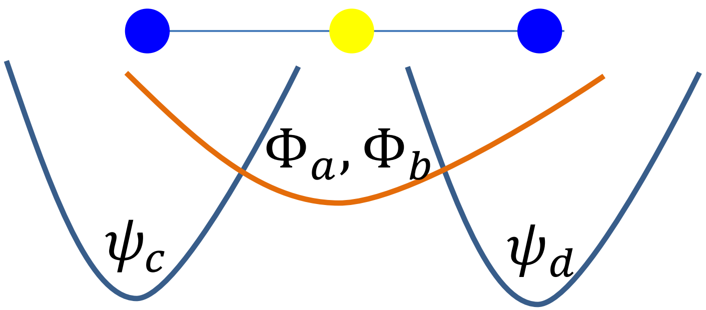
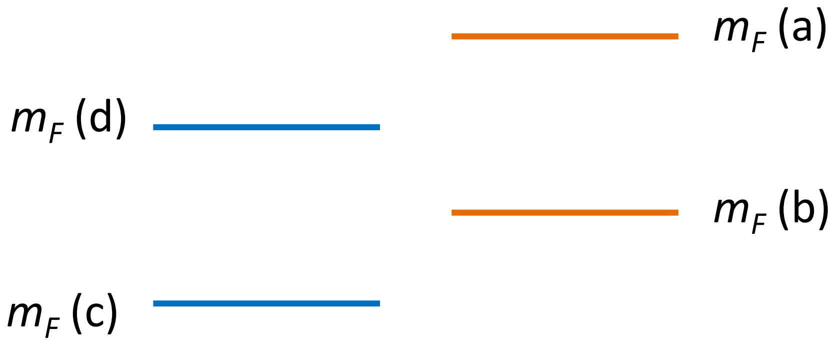
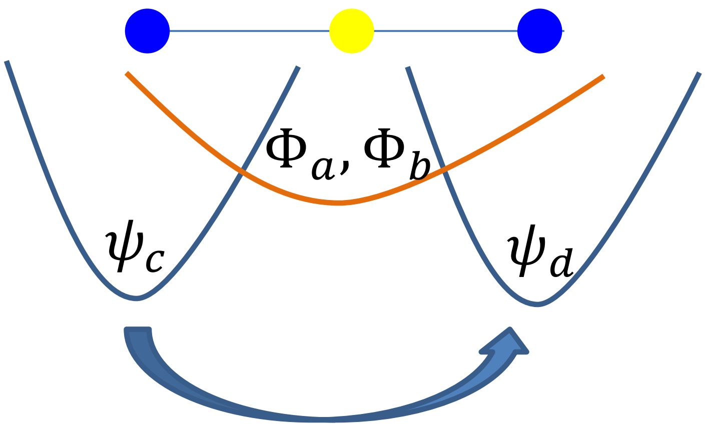


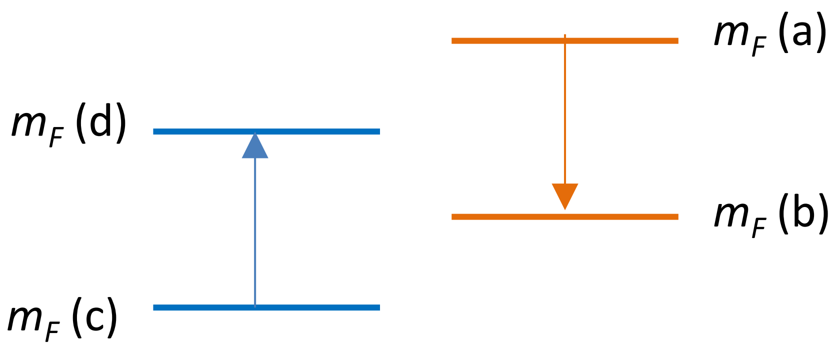
$$\psi_c^\dagger \, \Phi_a^\dagger \, \Phi_b \psi_d + \psi_d^\dagger \, \Phi_b^\dagger \, \Phi_a \psi_c$$



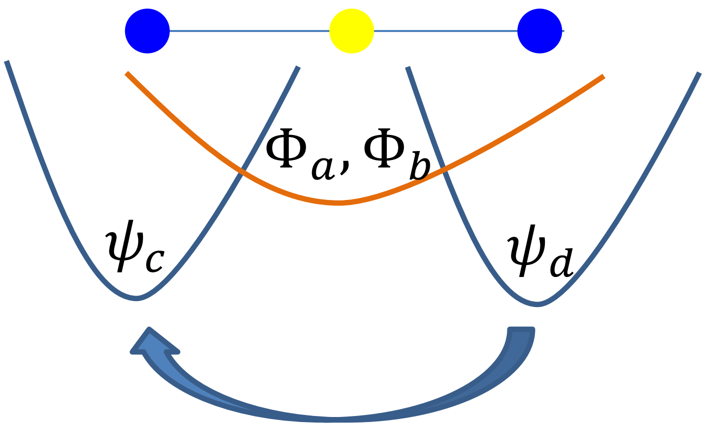
cQED LINK



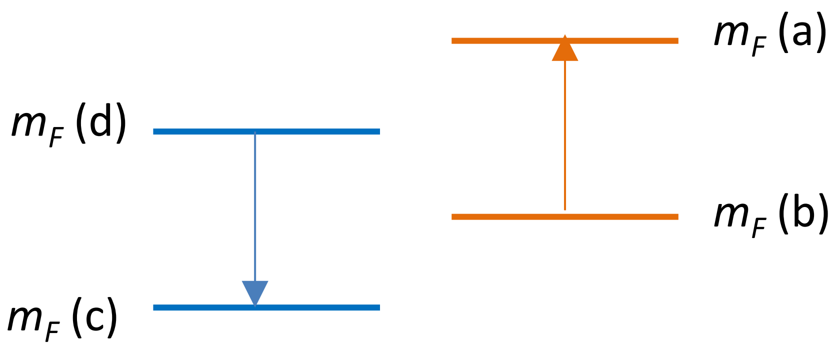
$$\psi_c^\dagger \, \Phi_a^\dagger \, \Phi_b \psi_d + \psi_d^\dagger \, \Phi_b^\dagger \, \Phi_a \psi_c$$



cQED LINK



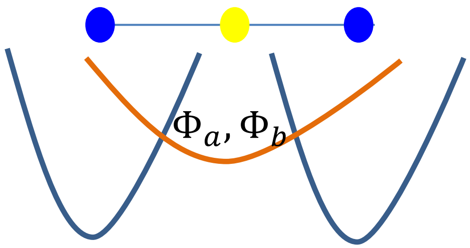
$$\psi_c{}^\dagger\ \Phi_a{}^\dagger\ \Phi_b\psi_d + \psi_d{}^\dagger\ \Phi_b{}^\dagger\ \Phi_a\psi_c$$



cQED LINK

$$L_+ = {\Phi_a}^\dagger \, \Phi_b \quad ; L_- = {\Phi_b}^\dagger \, \Phi_a$$

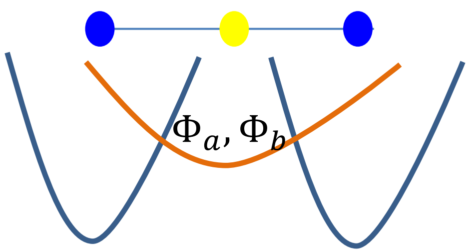
$$L_z = \frac{1}{2}(N_a - N_b) \quad ; l = \frac{1}{2}(N_a + N_b)$$



cQED LINK

$$L_{+} = \Phi_a{}^{\dagger}\,\Phi_b \quad ; L_{-} = \Phi_b{}^{\dagger}\,\Phi_a$$

$$L_z = \frac{1}{2}(N_a - N_b) \quad ; l = \frac{1}{2}(N_a + N_b)$$



and thus what we have is

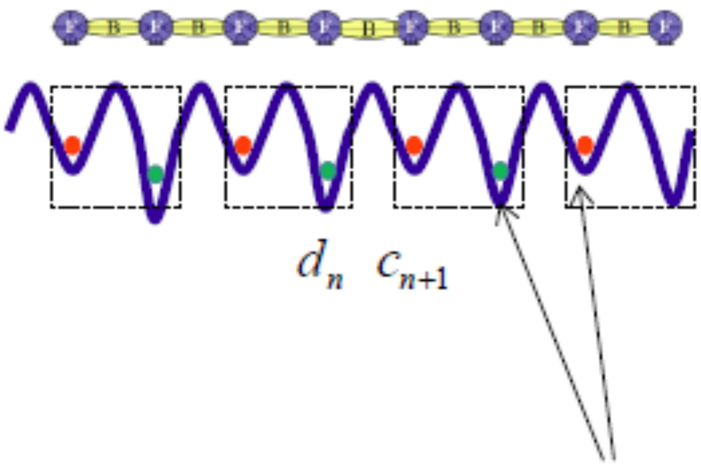
$$\psi_c^{\dagger}\Phi_a{}^{\dagger}\,\Phi_b\psi_d + \psi_d^{\dagger}\Phi_b{}^{\dagger}\,\Phi_a\psi_c$$

$$\psi_c^{\dagger}L_{+}\psi_d \sim \psi_c^{\dagger}e^{i\theta}\psi_d$$

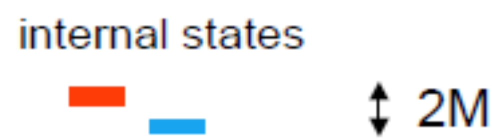
where for large l , $m \ll l$

$$L_{+} \sim e^{i(\phi_1 - \phi_2)} \equiv e^{i\theta}$$

DYNAMICAL FERMIONS



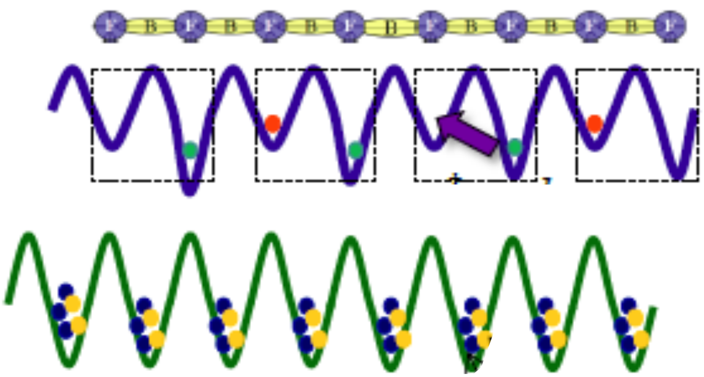
$$H_M = M\sum_n (-1)^n \psi_n^\dagger \psi_n$$

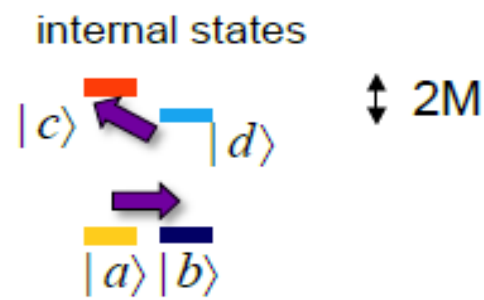


$$\{c_n, c_n^\dagger\} = \{d_n, d_n^\dagger\} = 1$$

Staggered Fermions”
L. Susskind Phys. Rev. D 16, 3031 (1977)

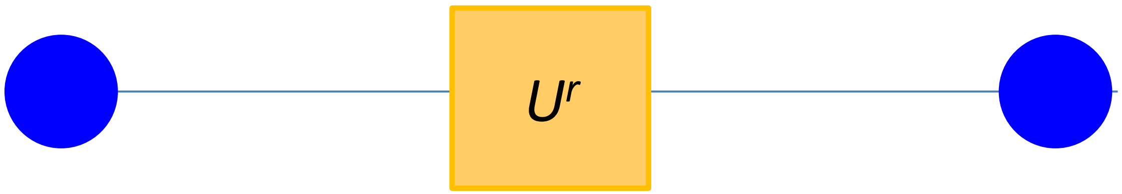
DYNAMICAL FERMIONS
SHWINGER MODEL





$$\frac{\epsilon}{\sqrt{\ell\left(\ell+1\right)}}\sum_{n}\left(\psi_{n}^{\dagger}L_{+,n}\psi_{n+1}+h.c.\right)$$

NON-ABELIAN LINK

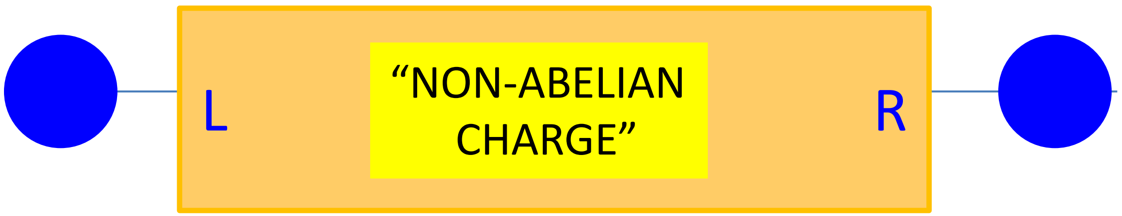


$$\psi_{\mathbf{n}} = (\psi_{\mathbf{n},a}) = \begin{pmatrix} \psi_{\mathbf{n},1} \\ \psi_{\mathbf{n},2} \\ \cdots \end{pmatrix}$$

U^r = element of the gauge group

$$H_{int} = \epsilon \sum_{\mathbf{n},k} \left(\psi_{\mathbf{n}}^{\dagger} U_{\mathbf{n},k}^{r} \psi_{\mathbf{n}+\hat{\mathbf{k}}} + h.c. \right)$$

NON-ABELIAN LINKS



$$H_{link} = \{|j, m, m'\rangle\} = \bigoplus_j \; [j_L \otimes j_R]_{maxE}$$

$$[L_a, R_b] = 0$$

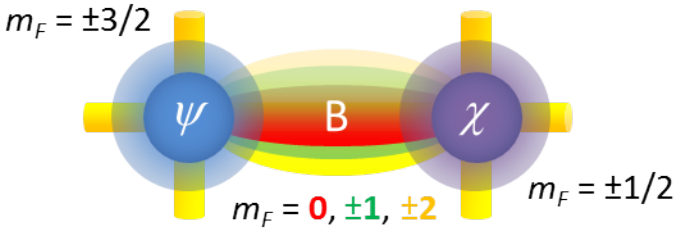
$$[L_a, U^r] = T^r_a U^r \quad ; \quad [R_a, U^r] = U^r T^r_a$$

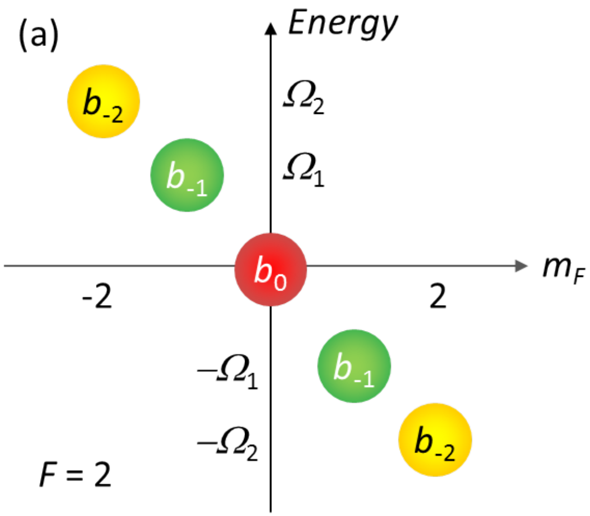
$$[L_a, L_b] = -i f_{abc} L_c \quad ; \quad [R_a, R_b] = i f_{abc} R_c \quad ;$$

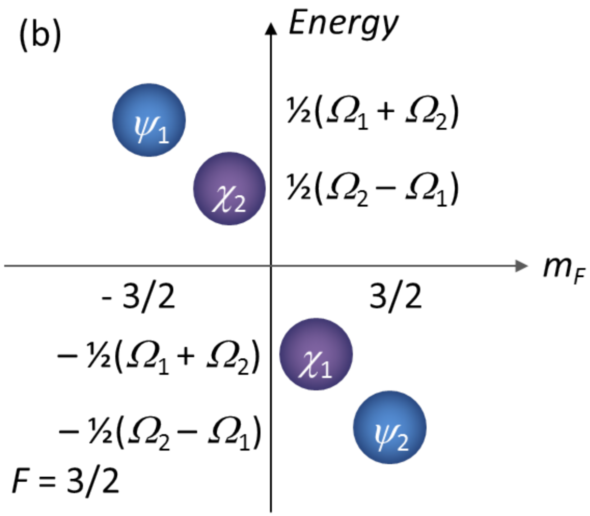
$$\sum_a L_a L_a = \sum_a R_a R_a \equiv \sum_a E_a E_a$$

SU(2) EXACT

$$H_{link} = 0 \oplus (\frac{1}{2} \otimes \frac{1}{2})$$



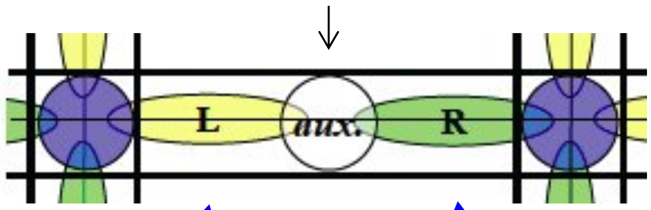




SU(2) EFFECTIVE

Ancillary "constraint" Fermion

"color" fermions

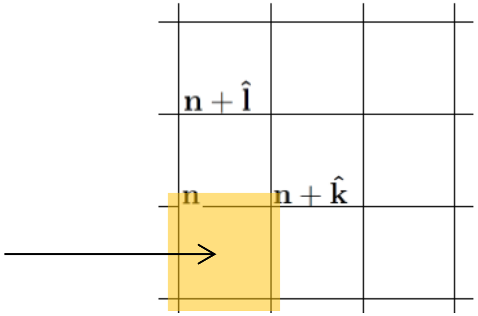


$$U_L = \frac{1}{\sqrt{N_L+1}} \begin{pmatrix} a_1^\dagger & -a_2 \\ a_2^\dagger & a_1 \end{pmatrix} \; ; \; U_R = \begin{pmatrix} b_1^\dagger & b_2^\dagger \\ -b_2 & b_1 \end{pmatrix} \frac{1}{\sqrt{N_R+1}}$$

$$U = U_L U_R$$

On each link – $a_{1,2}$ bosons on the left, $b_{1,2}$ bosons on the right

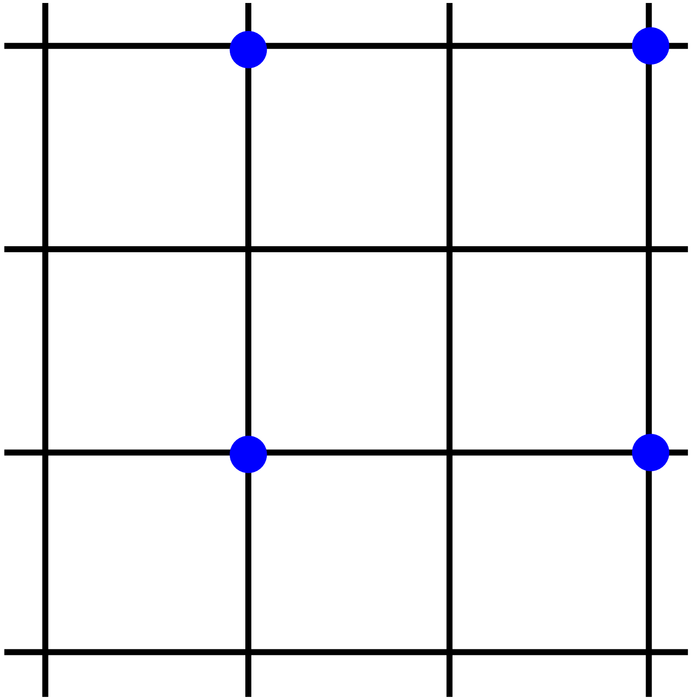
PLAQUETTES



PLAQUETTES

1d elementary link interactions – **already gauge invariant building blocks** of effective plaquettes

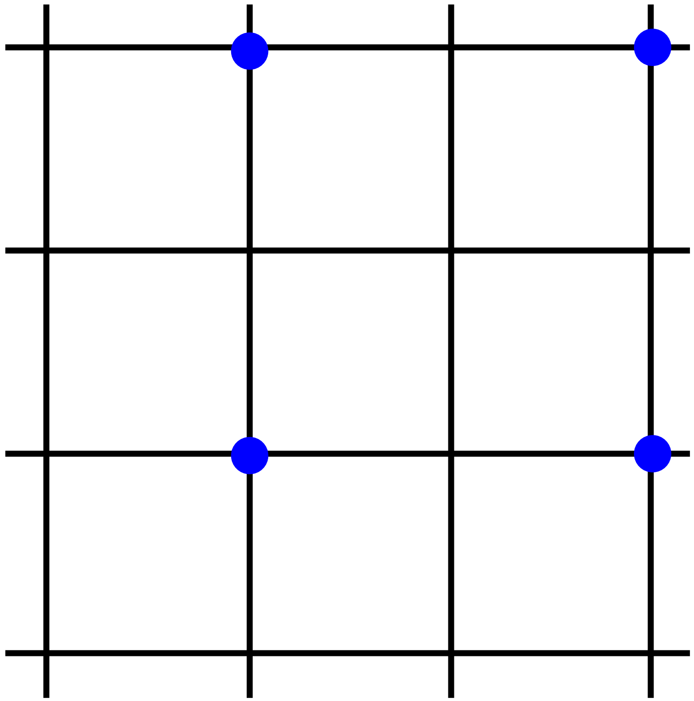
Auxiliary fermions := ●



PLAQUETTES

1d elementary link interactions – **already gauge invariant building blocks** of effective plaquettes

Auxiliary fermions := ●

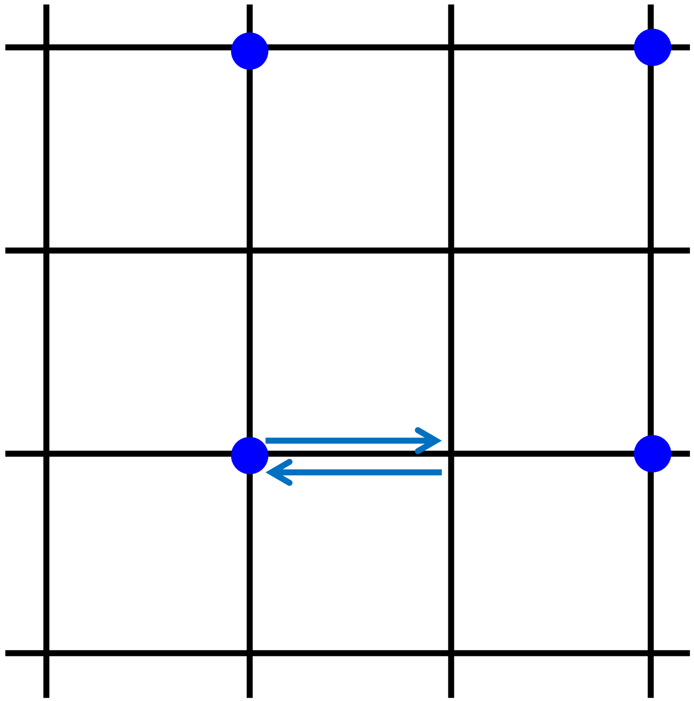


PLAQUETTES

1d elementary link interactions – **already gauge invariant building blocks** of effective plaquettes

Auxiliary fermions

– virtual processes

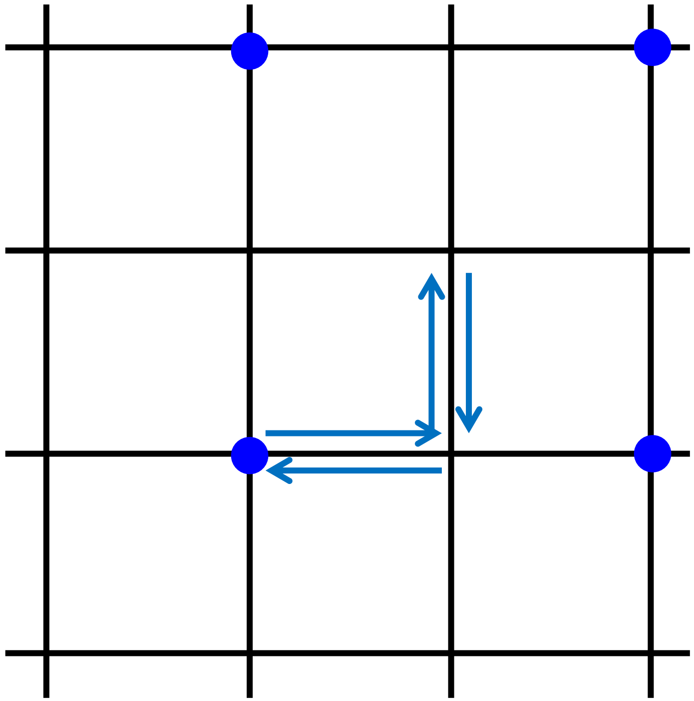


PLAQUETTES

1d elementary link interactions – **already gauge invariant building blocks** of effective plaquettes

Auxiliary fermions

– virtual processes



PLAQUETTES

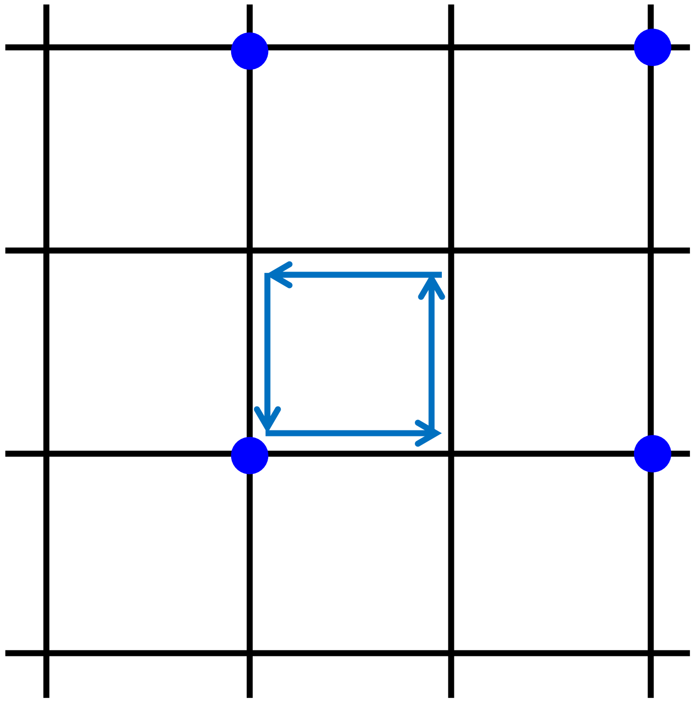
1d elementary link interactions – **already gauge invariant building blocks** of effective plaquettes

Auxiliary fermions

– virtual processes

- plaquettes.

$$\sum_{\text{plaquettes}} \left(\mathrm{Tr}\left(U_1 U_2 U_3^{\dagger} U_4^{\dagger} \right) + h.c. \right)$$



PLAQUETTES

1d elementary link interactions – **already gauge invariant building blocks** of effective plaquettes

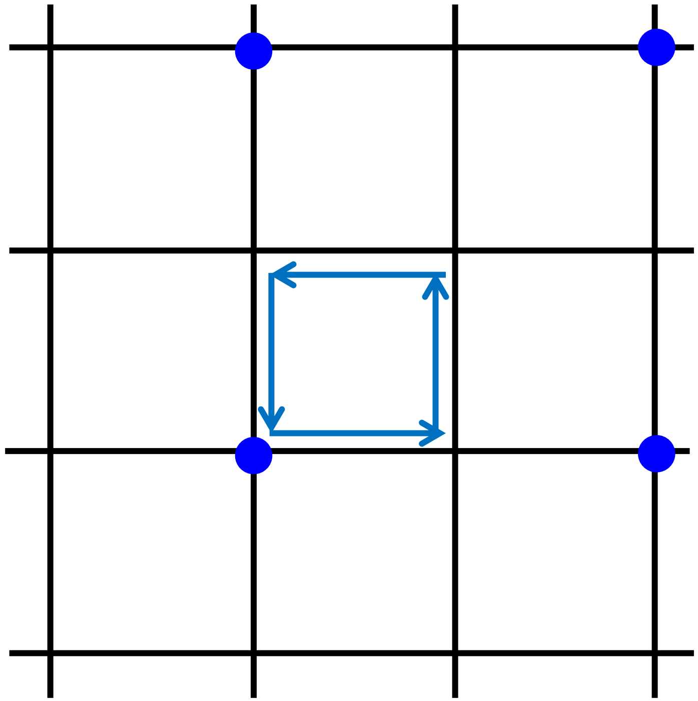
Auxiliary fermions

– virtual processes

- plaquettes.

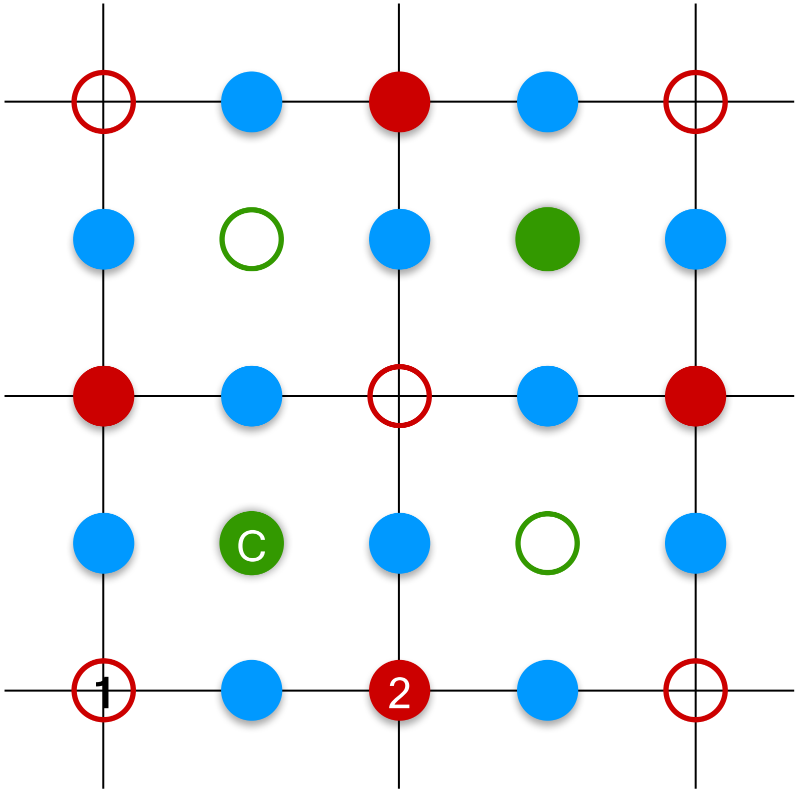
$$\sum_{\text{plaquettes}} \left(\text{Tr} \left(U_1 U_2 U_3^\dagger U_4^\dagger \right) + h.c. \right)$$

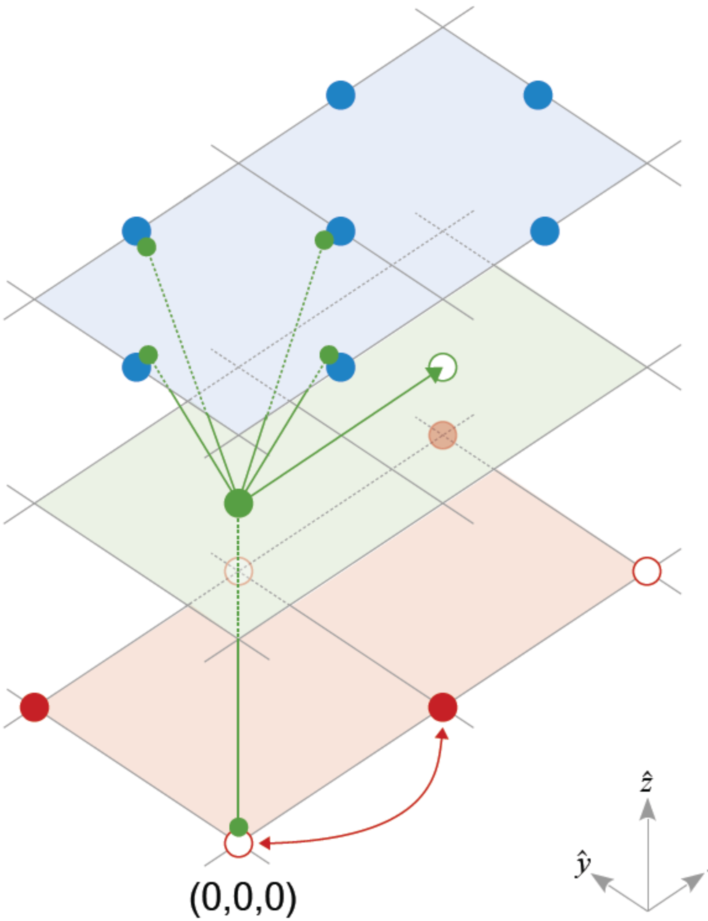
OKAY for: discrete, abelian & non-abelian groups



DIGITAL SIMULATION

Three atomic layers w. movable control atoms

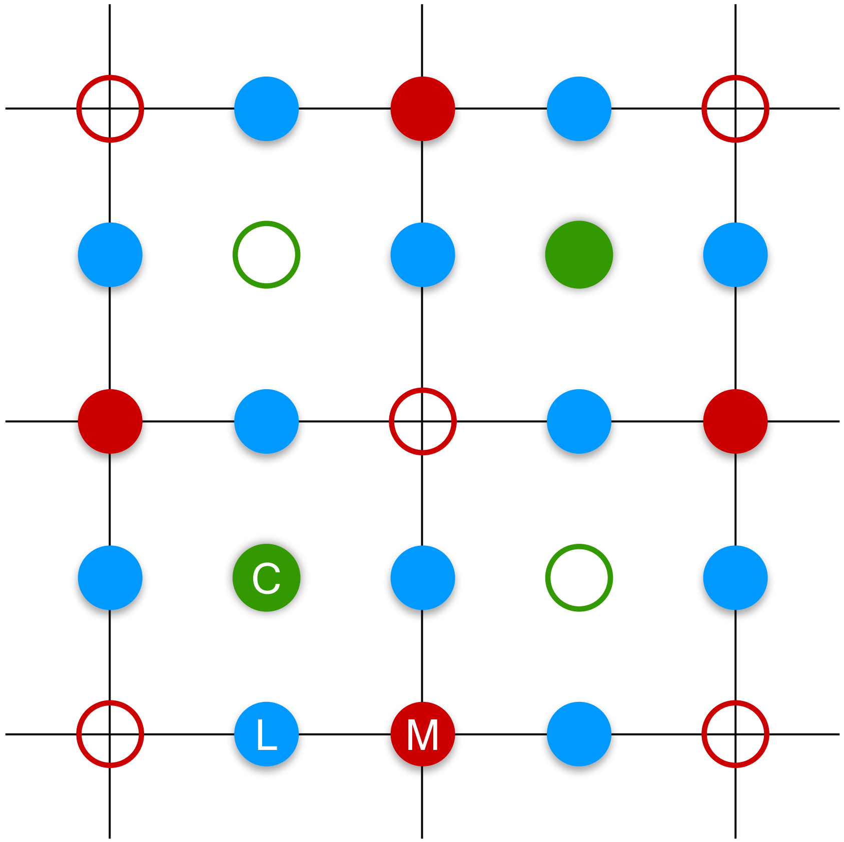




E. Zohar, A. Farace, B. Reznik, J. I. Cirac, PRA 2017.

E. Zohar, A. Farace, B. Reznik, J. I. Cirac, PRL 2017.

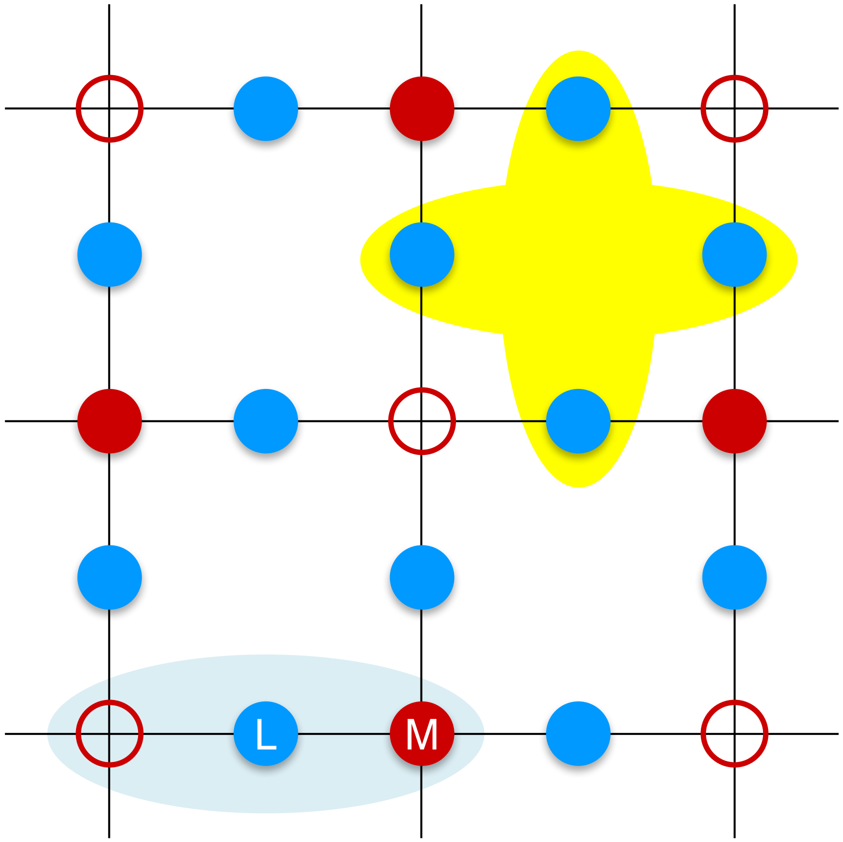
Lattice Gauge Theory with Stators



Matter Fermions
Link (Gauge) degrees of freedom
Control degrees of freedom

E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. A 2017.
E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. Lett. 2017.

Digital Lattice Gauge Theories



The Z_2 example:

- Plaquette interactions

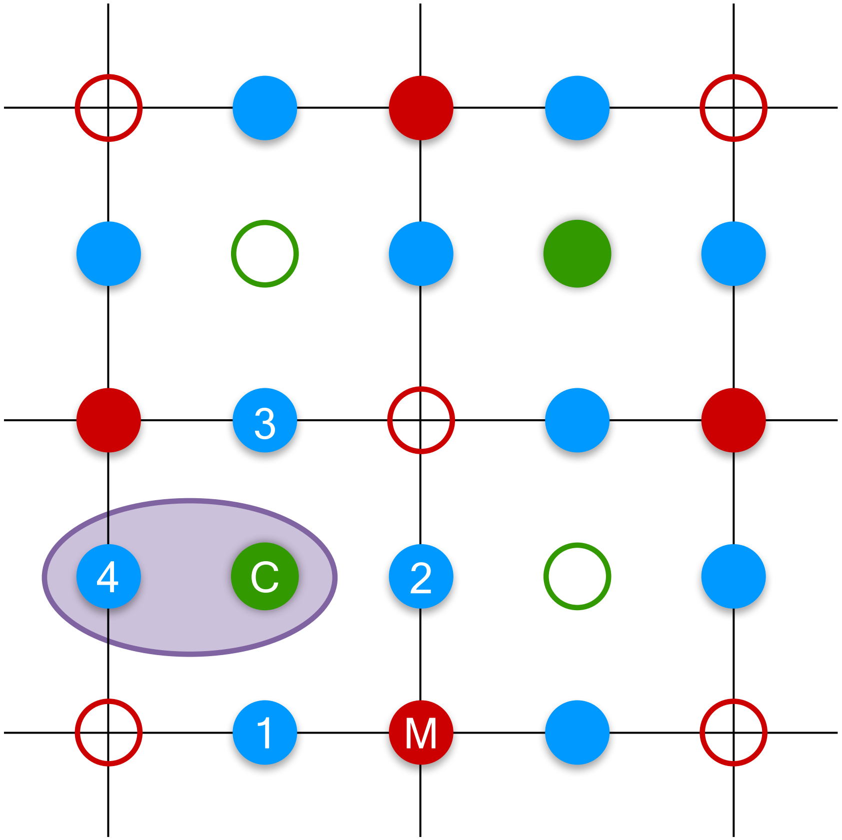
$$\sigma_x(\mathbf{x},1)\,\sigma_x(\mathbf{x}+\hat{\mathbf{1}},2)\,\sigma_x(\mathbf{x}+\hat{\mathbf{2}},1)\,\sigma_x(\mathbf{x},2)$$

- Link interactions

$$\psi^\dagger(\mathbf{x})\sigma_x(\mathbf{x},k)\psi(\mathbf{x}+\hat{\mathbf{k}})$$

Plaquettes: Four-body Interactions

Stators: two-body interactions → four-body interactions



$$\left|\widetilde{in}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|\widetilde{\uparrow}\right\rangle + \left|\widetilde{\downarrow}\right\rangle\right)$$

$$\mathcal{U}_4^\dagger \left|\widetilde{in}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|\widetilde{\uparrow}\right\rangle + \sigma_4^x \otimes \left|\widetilde{\downarrow}\right\rangle\right)$$

Plaquettes: Four-body Interactions

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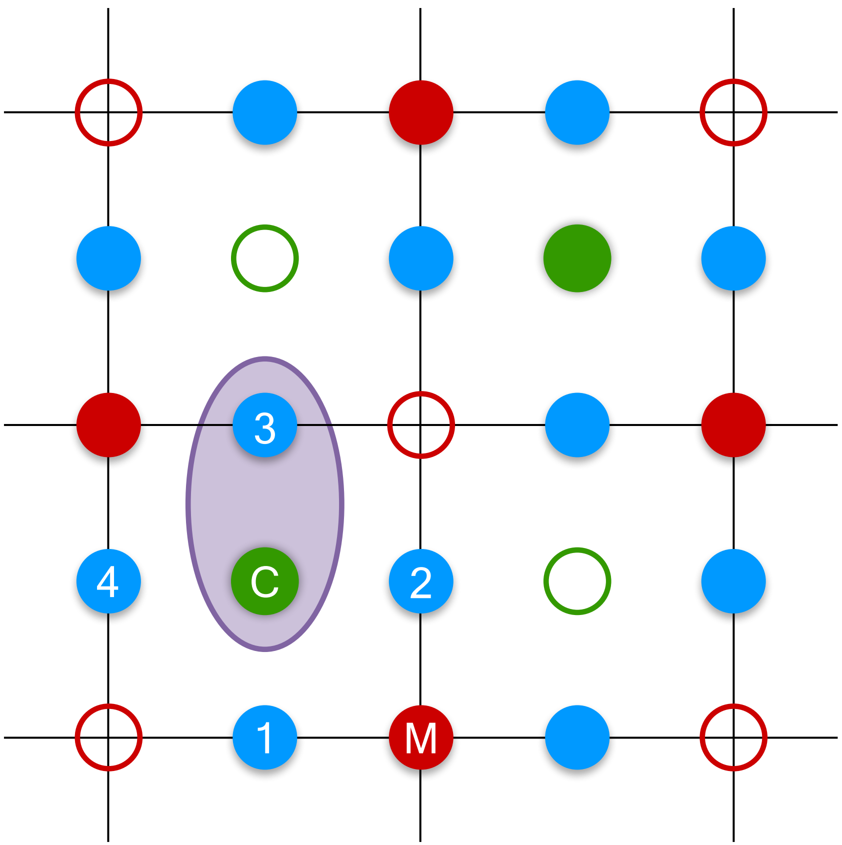

$$|\widetilde{in}\rangle = \frac{1}{\sqrt{2}}\left(|\tilde{\uparrow}\rangle + |\tilde{\downarrow}\rangle\right)$$

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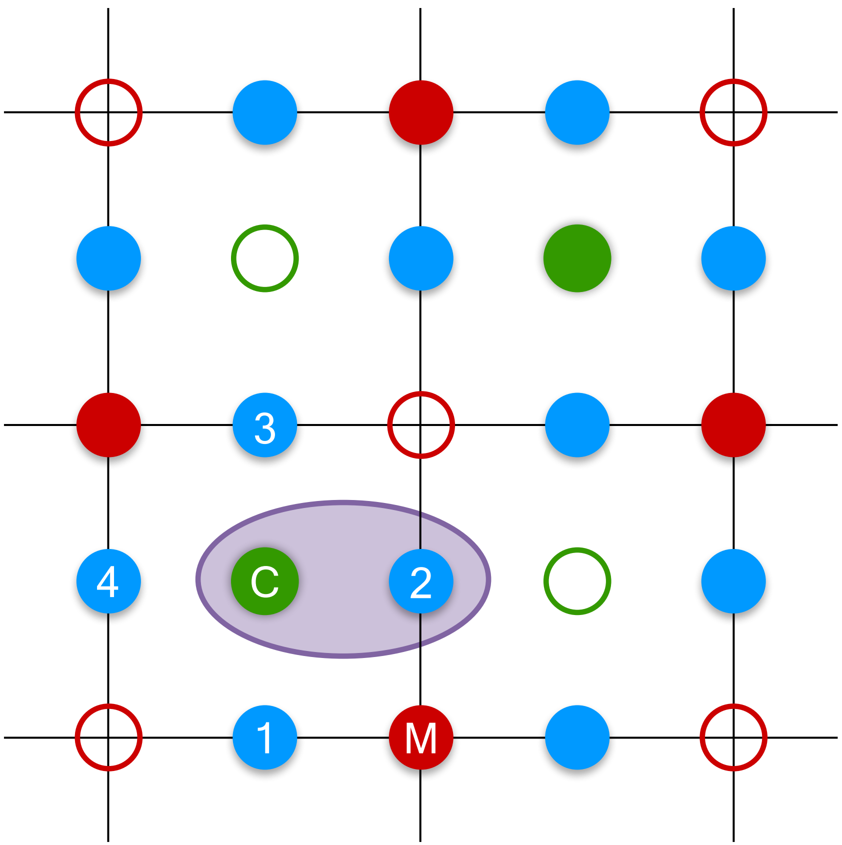

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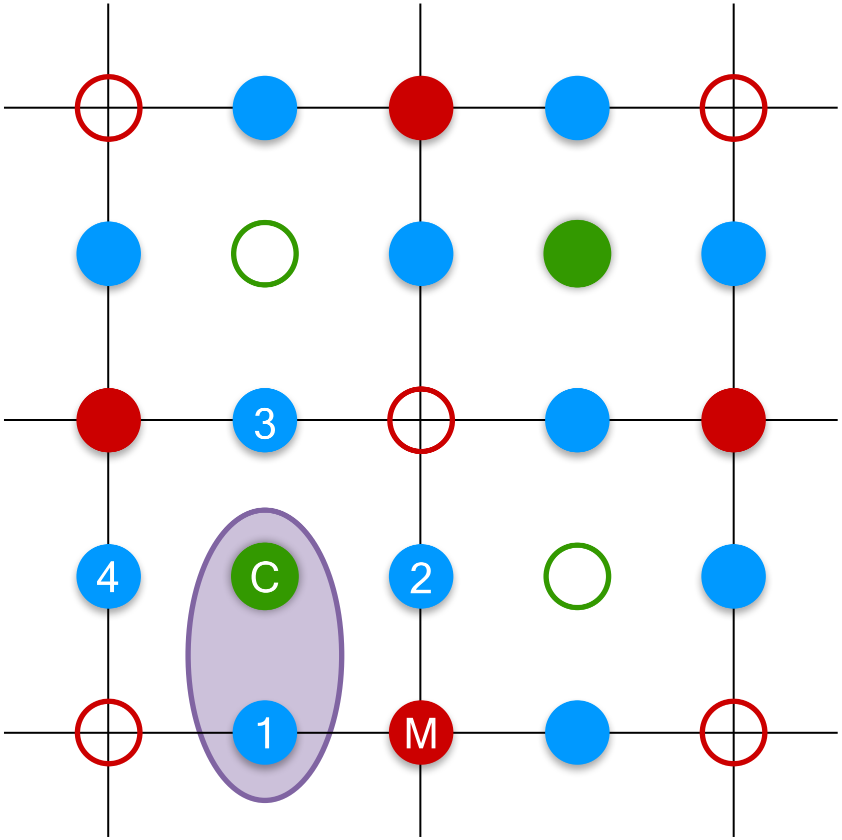
$$\mathcal{U}_3^\dagger \mathcal{U}_4^\dagger |\widetilde{in}\rangle = \frac{1}{\sqrt{2}}\left(|\widetilde{\uparrow}\rangle + \sigma_3^x \sigma_4^x \otimes |\widetilde{\downarrow}\rangle\right)$$

$$\mathcal{U}_2 \mathcal{U}_3^\dagger \mathcal{U}_4^\dagger |\widetilde{in}\rangle = \frac{1}{\sqrt{2}}\left(|\widetilde{\uparrow}\rangle + \sigma_2^x \sigma_3^x \sigma_4^x \otimes |\widetilde{\downarrow}\rangle\right)$$

Plaquettes: Four-body Interactions

Stators: two-body interactions → four-body interactions



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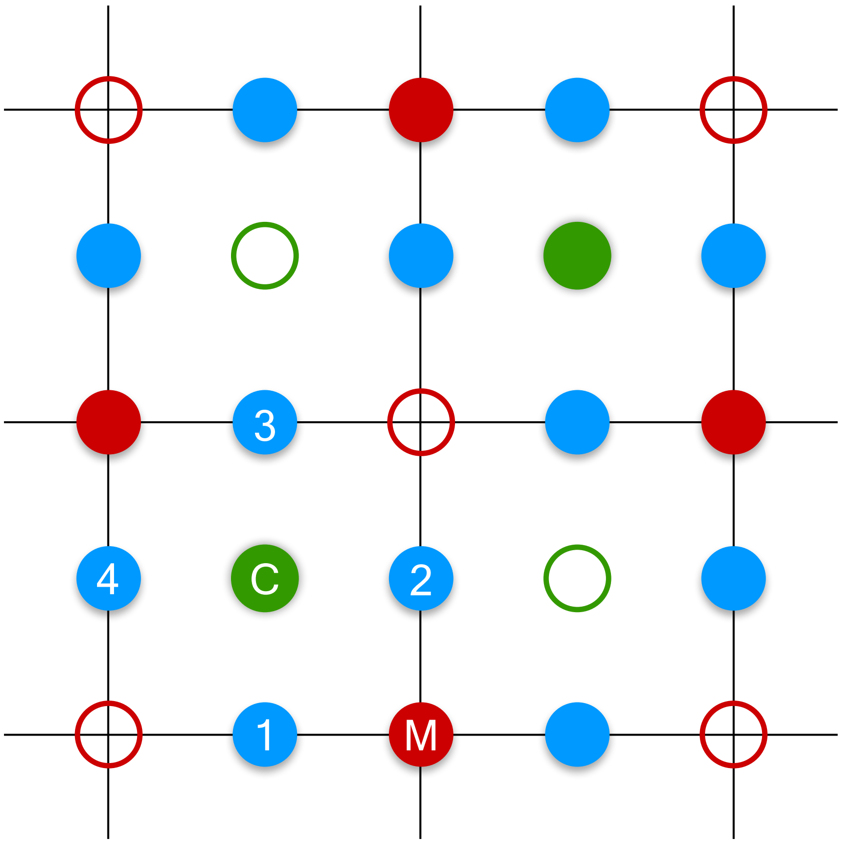
$$\mathcal{U}_3^\dagger\mathcal{U}_4^\dagger\left|\widetilde{in}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|\widetilde{\uparrow}\right\rangle + \sigma_3^x\sigma_4^x \otimes \left|\widetilde{\downarrow}\right\rangle\right)$$

$$\mathcal{U}_2\mathcal{U}_3^\dagger\mathcal{U}_4^\dagger\left|\widetilde{in}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|\widetilde{\uparrow}\right\rangle + \sigma_2^x\sigma_3^x\sigma_4^x \otimes \left|\widetilde{\downarrow}\right\rangle\right)$$

$$\mathcal{U}_1\mathcal{U}_2\mathcal{U}_3^\dagger\mathcal{U}_4^\dagger\left|\widetilde{in}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|\widetilde{\uparrow}\right\rangle + \sigma_1^x\sigma_2^x\sigma_3^x\sigma_4^x \otimes \left|\widetilde{\downarrow}\right\rangle\right)$$

Plaquettes: Four-body Interactions

Stators: two-body interactions → four-body interactions



$$\left|\widetilde{in}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|\widetilde{\uparrow}\right\rangle + \left|\widetilde{\downarrow}\right\rangle\right)$$

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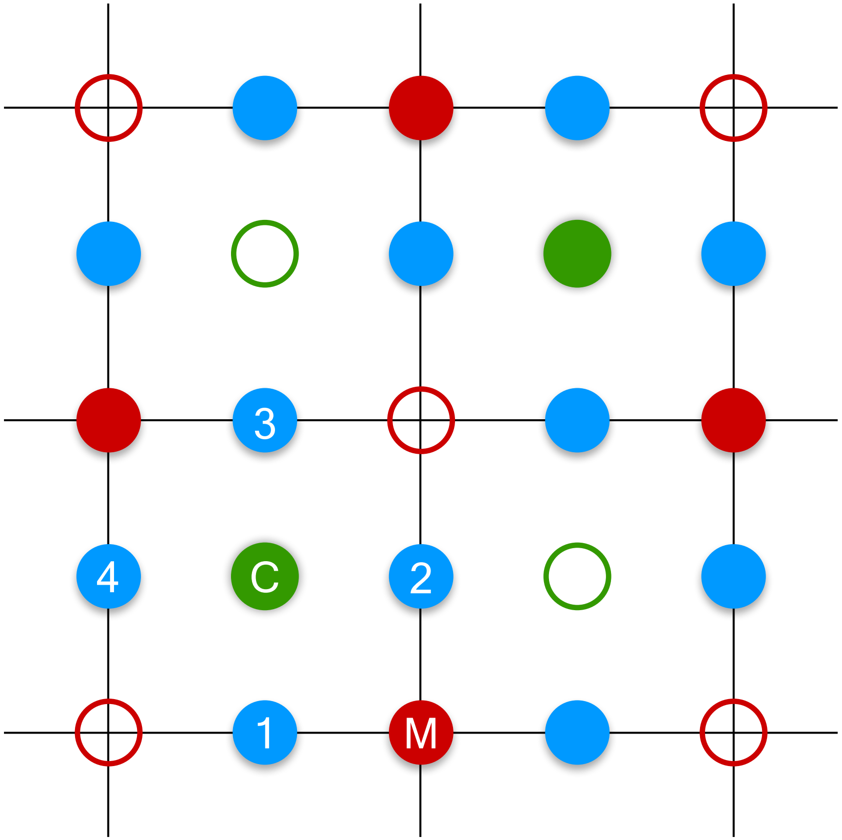
$$\mathcal{U}_1\mathcal{U}_2\mathcal{U}_3^\dagger\mathcal{U}_4^\dagger\left|\widetilde{in}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|\widetilde{\uparrow}\right\rangle + \sigma_1^x\sigma_2^x\sigma_3^x\sigma_4^x \otimes \left|\widetilde{\downarrow}\right\rangle\right)$$

$$S_\square = \frac{1}{\sqrt{2}}\left(\left|\widetilde{\uparrow}\right\rangle + \sigma_\square^x \otimes \left|\widetilde{\downarrow}\right\rangle\right)$$

Plaquettes: Four-body Interactions

Stators: two-body interactions → four-body interactions



$$S_{\square} = \frac{1}{\sqrt{2}}\left(\left|\widetilde{\uparrow}\right\rangle + \sigma^x_{\square} \otimes \left|\widetilde{\downarrow}\right\rangle\right)$$

$$\widetilde{\sigma}^x S_{\square} = S_{\square}\sigma^x_{\square}$$

$$e^{-i\lambda\widetilde{\sigma}^x\tau} S_{\square} = S_{\square} e^{-i\lambda\sigma^x_{\square}\tau}$$

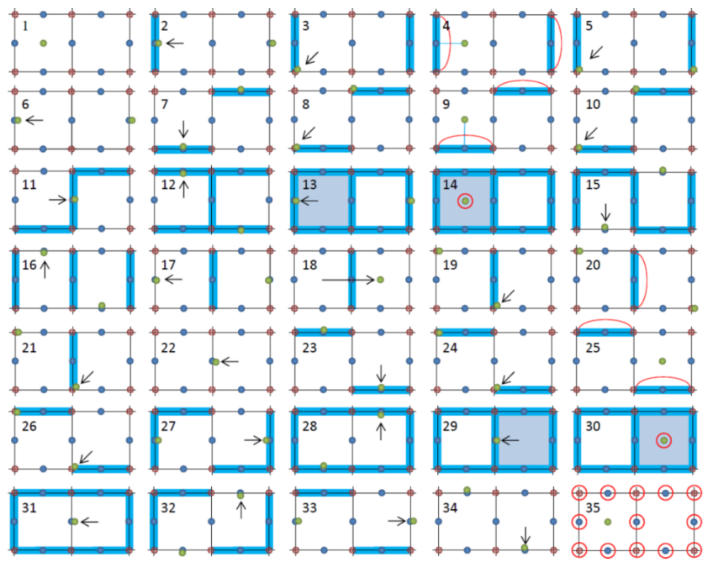
$$\mathcal{U}_4\mathcal{U}_3\mathcal{U}_2^\dagger\mathcal{U}_1^\dagger e^{-i\lambda\widetilde{\sigma}^x\tau}\mathcal{U}_1\mathcal{U}_2\mathcal{U}_3^\dagger\mathcal{U}_4^\dagger\left|\widetilde{in}\right\rangle = \left|\widetilde{in}\right\rangle e^{-i\lambda\sigma^x_{\square}\tau}$$

DIGITAL SIMULATION

A bipartite single time step

Trotterized time evolution, of already gauge invariants elements

$$e^{-i\Sigma_j H_j t} = \lim_{M\to\infty} \left(\Pi_j e^{-iH_j \frac{t}{M}}\right)^M$$



$$\left\| \left(\tilde{\mathfrak{U}}_M^{(2)}(t) - \mathfrak{U}(t) \right) \right\| \leq 60 \frac{t^3 L^6 \lambda_{max}^3}{M^2}.$$

QUANTUM SIMULATIONS

COLD ATOMS – EXPERIMENTS

PRL **103**, 080404 (2009) PHYSICAL REVIEW LETTERS week ending 21 AUGUST 2009

Experimental Demonstration of Single-Site Addressability in a Two-Dimensional Optical Lattice

Peter Würtz,[1] Tim Langen,[1] Tatjana Gericke,[1] Andreas Koglbauer,[1] and Herwig Ott[1,2,*]

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[2]*Research Center OPTIMAS, Technische Universität Kaiserslautern, 67663 Kaiserslautern, Germany*

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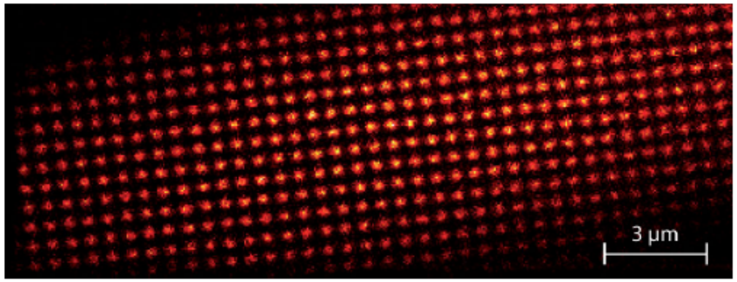


FIG. 1 (color online). Electron microscope image of a Bose-Einstein condensate in a 2D optical lattice with 600 nm lattice spacing (sum obtained from 260 individual experimental realizations). Each site has a tubelike shape with an extension of 6 μm perpendicular to the plane of projection. The central lattice sites contain about 80 atoms.

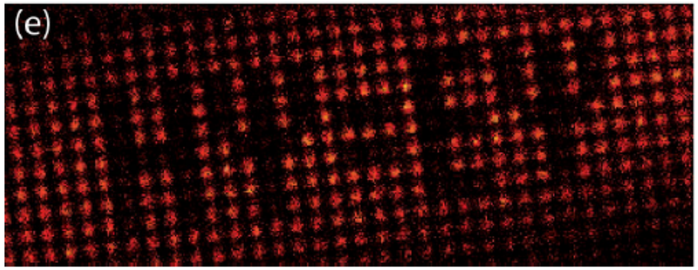


FIG. 2 (color online). Patterning a Bose-Einstein condensate in a 2D optical lattice with a spacing of 600 nm. Every emptied site was illuminated with the electron beam (7 nA beam current, 100 nm FWHM beam diameter) for (a),(b) 3, (c),(d) 2, and (e) 1.5 ms, respectively. The imaging time was 45 ms. Between 150 and 250 images from individual experimental realizations have been summed for each pattern.

QUANTUM SIMULATIONS
COLD ATOMS – EXPERIMENTS

nature | Vol 462 | 5 November 2009 | doi:10.1038/nature08482

A quantum gas microscope for detecting single atoms in a Hubbard-regime optical lattice

Waseem S. Bakr[1], Jonathon I. Gillen[1], Amy Peng[1], Simon Fölling[1] & Markus Greiner[1]

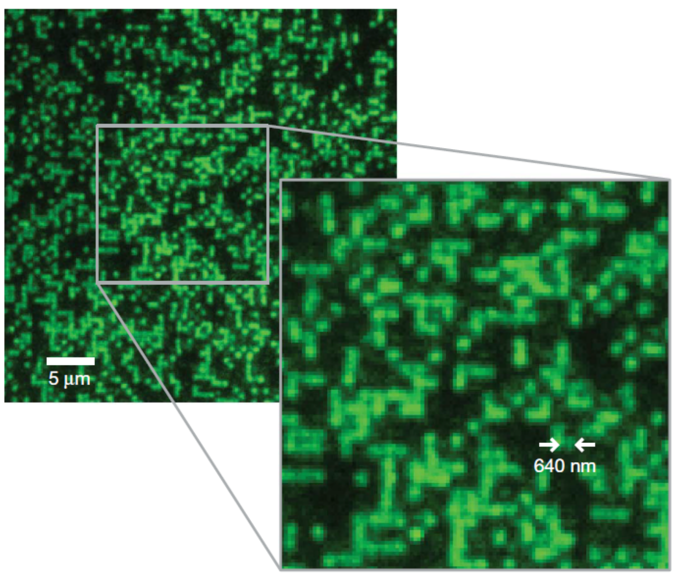


Figure 3 | Site-resolved imaging of single atoms on a 640-nm-period optical lattice, loaded with a high density Bose–Einstein condensate. Inset, magnified view of the central section of the picture. The lattice structure and the discrete atoms are clearly visible. Owing to light-assisted collisions and molecule formation on multiply occupied sites during imaging, only empty and singly occupied sites can be seen in the image.

mined through preparation and measurement. By implementing a high-resolution optical imaging system, single atoms are detected with near-unity fidelity on individual sites of a Hubbard-regime optical lattice. The lattice itself is generated by projecting a holographic mask through the imaging system. It has an arbitrary geometry, chosen to support both strong tunnel coupling between lattice sites and strong on-site confinement. Our approach can be

QUANTUM SIMULATIONS

COLD ATOMS – EXPERIMENTS

doi:10.1038/nature09827

Single-spin addressing in an atomic Mott insulator

Christof Weitenberg[1], Manuel Endres[1], Jacob F. Sherson[1]†, Marc Cheneau[1], Peter Schauß[1], Takeshi Fukuhara[1], Immanuel Bloch[1,2] & Stefan Kuhr[1]

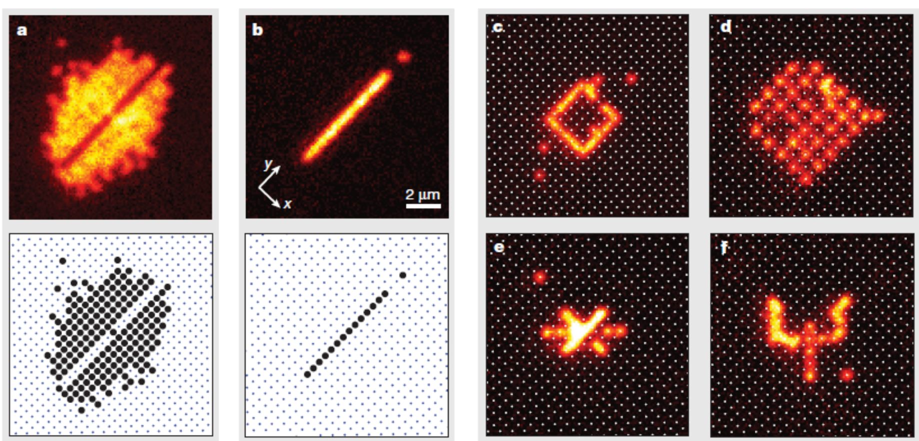
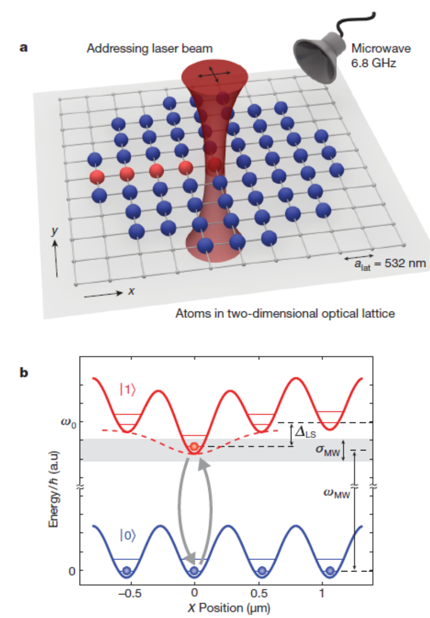


Figure 2 | Single-site addressing. **a**, Top, experimentally obtained fluorescence image of a Mott insulator with unity filling in which the spin of selected atoms was flipped from |0⟩ to |1⟩ using our single-site addressing scheme. Atoms in state |1⟩ were removed by a resonant laser pulse before detection. Bottom, the reconstructed atom number distribution on the lattice. Each filled circle indicates a single atom; the points mark the lattice sites. **b**, Top, as for **a** except that a global microwave sweep exchanged the population in |0⟩ and |1⟩, such that only the addressed atoms were observed. Bottom, the reconstructed atom number distribution shows 14 atoms on neighbouring sites. **c–f**, As for **b**, but omitting the atom number distribution. The images contain 29 (**c**), 35 (**d**), 18 (**e**) and 23 (**f**) atoms. The single isolated atoms in **b**, **e** and **f** were placed intentionally to allow for the correct determination of the lattice phase for the feedback on the addressing beam position.



QUANTUM SIMULATIONS

IONS – EXPERIMENTS

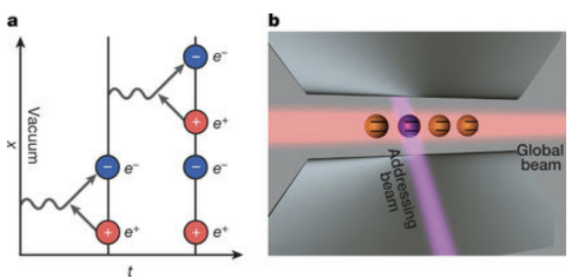
LETTER

doi:10.1038/nature18318

Real-time dynamics of lattice gauge theories with a few-qubit quantum computer

Esteban A. Martinez[1]*, Christine A. Muschik[2,3]*, Philipp Schindler[1], Daniel Nigg[1], Alexander Erhard[1], Markus Heyl[2,4], Philipp Hauke[2,3], Marcello Dalmonte[2,3], Thomas Monz[1], Peter Zoller[2,3] & Rainer Blatt[1,2]

Figure 1: Quantum simulation of the Schwinger mechanism.



a, The instability of the vacuum due to quantum fluctuations is one of the most fundamental effects in gauge theories. We simulate the coherent real-time dynamics of particle–antiparticle creation by realizing the Schwinger model (one-

systems. In contrast, quantum simulations aim at the long-term goal of solving the specific yet fundamental class of problems that currently cannot be tackled by these classical techniques. The digital approach we employ here is based on the Hamiltonian formulation of gauge theories[9], and enables direct access to the system wavefunction. As we show below, this allows us to investigate entanglement generation during particle–antiparticle production, emphasizing a novel perspective on the dynamics of the Schwinger mechanism[2].

QUANTUM SIMULATIONS
COLD ATOMS – EXPERIMENTS

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PAPER

Implementing quantum electrodynamics with ultracold atomic systems

V Kasper[1,2], F Hebenstreit[3], F Jendrzejewski[4], M K Oberthaler[4] and J Berges[1]

Oberthaler group

QUANTUM SIMULATIONS
COLD ATOMS – EXPERIMENTS

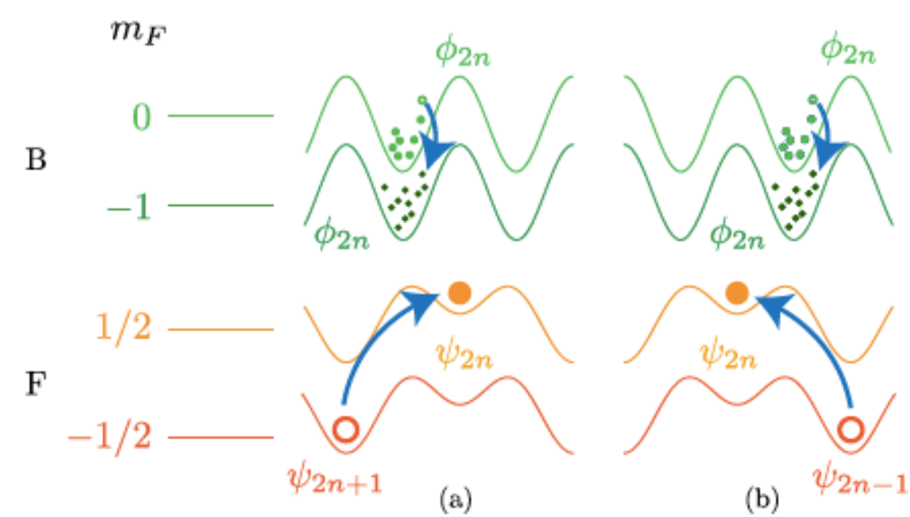

Figure 4. The selection procedure results in the correlated bosonic spin exchange with a fermionic hopping in the superlattice. Note that the inverse process is allowed as well.

4. Microscopic parameters

At this point, we are now able to determine the accessible parameters for an experimental implementation of the Schwinger model via a mixture of bosonic ^{23}Na and fermionic ^{6}Li atoms [30], which is determined by the parameters χ_{BB}, χ_{BF}, Δ and the occupation numbers of the links.

CONFINEMENT
TOY MODELS

- 1+1D: Schwinger's model.

- cQED: 2+1D: no phase transition
  Instantons give rise to confinement at $g < 1$ (Polyakov).
  (For T > 0: there is a phase transition also in 2+1D.)

- cQED: 3+1D: phase transition between a strong coupling confining phase, and a weak coupling coulomb phase.

- Z(N): for $\mathrm{N} \geq N_c$: Three phases: electric confinement, magnetic confinement, and non confinement.

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Hep simulations/GROUPS

– ICFO, Barcelona (Lewenstein)
– Innsbruck (Zoller, Blatt)
– University of Bern (Wiese)
– Heidelberg (Oberthaler, Berges)
– UGENT (Verstraete)
– Caltech, UMD (Preskill, Jordan)
– …
– Tensor Networks with LGI (cQED in 1+1, 2+1), MPQ,UGENT,Ulm,Mainz

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Thank You!