SIMULATING LATTICE GAUGE THEORIES WITH COLD ATOMS

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MPQ

Workshop on Computational Complexity and High Energy Physics
August 2017
University of Maryland
OUTLINE

QUANTUM SIMULATION
COLD ATOMS IN OPTICAL LATTICES
HAMILTONIAN LATTICE GAUGE THEORY (LGT)

ANALOG SIMULATION: LGT
– REQUIREMENTS
– EXACT AND EFFECTIVE LOCAL GAUGE INVARIANCE
– LINKS AND PLAQUETTES – Examples: cQED, SU(2)
DIGITAL SIMULATION

CURRENT EXPERIMENTS
OUTLOOK.
QUANTUM SIMULATION
ANALOG

PHYSICAL SYSTEM

(Q Phenomenological) Hamiltonian

\[ H = \ldots \]

QUANTUM SIMULATOR

Physical Hamiltonian

\[ H = \ldots \]


- **Control:** External fields

- **Trapping:**
  - Lasers
  - Magnetic fields

- **Cooling:**
  - Lasers
  - Evaporation

- **Internal manipulation:**
  - Lasers
  - RF fields
  - Purification
  - Coherence
  - Detection

- **Interactions:**
  - Tune scattering length
COLD ATOMS

- Many-body phenomena
  - Degeneracy: bosons and fermions (BE/FD statistics)
  - Coherence: interference, atom lasers, four-wave mixing, ...
  - Superfluidity: vortices
  - Disorder: Anderson localization
  - Fermions: BCS-BEC

+ many other phenomena
Laser standing waves: dipole-trapping
In the presence $E(r, t)$ the atoms has a time dependent dipole moment $d(t) = \alpha(\omega) E(r, t)$ of some non resonant excited states. Stark effect:

$$V(r) \equiv \Delta E(r) = \alpha(\omega) \langle E(r, t) E(r, t) \rangle / \delta$$
(a) 2d array of effective 1d traps
(b) 3d square lattice

M. Lewenstein et. al, Advances in Physics, 2010.
COLD ATOMS
OPTICAL LATTICES

Laser standing waves: dipole-trapping

\[ H = \int \Psi_\sigma^\dagger \left( -\nabla^2 + V(r) \right) \Psi_\sigma + \hbar \omega \int \Psi_\sigma_1^\dagger \Psi_\sigma_2^\dagger \Psi_\sigma_3 \Psi_\sigma_4 \]

Lattice theory: Bose/Fermi-Hubbard model

\[ H = -t \sum_n (a_n^\dagger a_{n+1} + h.c.) + U \sum_n a_n^\dagger a_n^2 \]

Quantum phase transition from a superfluid to a Mott insulator in a gas of ultracold atoms

articles
COLD ATOMS
QUANTUM SIMULATIONS

Bosons/Fermions: \[ H = - \sum_{<n,m>, \sigma, \sigma'} (t_{\sigma, \sigma'} a_{n, \sigma}^\dagger a_{m, \sigma'} + h.c.) + \sum_{n, \sigma, \sigma'} U_{\sigma, \sigma'} a_{n, \sigma}^\dagger a_{n, \sigma'} a_{n, \sigma} a_{n, \sigma} \]

Spins: \[ H = - \sum_{<n,m>, \sigma, \sigma'} \left( J_x S_n^x S_m^x + J_y S_n^y S_m^y + J_z S_n^z S_m^z \right) + \sum_{n, \sigma, \sigma'} B_n S_n^z \]

CONDENSED MATTER PHYSICS
COLD ATOMS
QUANTUM SIMULATIONS

HIGH ENERGY PHYSICS?
24 < ORDERS OF MAGNITUDE!!
LONG RANGE FORCES?
LONG RANGE FORCES?
REQUIRE FORCE CARRIER

‘NEW FIELD’
REQUIRE FORCE CARRIER

'GAUGE FIELD'
THE STANDARD MODEL

• Matter: = fermions
  (Quarks and Leptons w.
   mass, spin 1/2, flavor, charge)

• Interactions mediators := YM gauge fields
  (spin 1 bosons).

  Electromagnetic: massless chargeless photon, (1), U(1)
  Weak interaction : massive, charged Z, W’s , (3), SU(2)
  Strong interaction : massless Gluons , (8), SU(3)
<table>
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<th>Abelian Fields</th>
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<td>Linear dynamics</td>
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We (ordinarily) don’t need QFT quantum field theory to understand the structure of atoms:

$$\alpha_{QED} \ll 1, \quad V_{QED}(r) \propto \frac{1}{r}$$

$$m_e c^2 \gg E_{Ry} \approx \alpha_{QED}^2 m_e c^2$$

But also higher energies effects are well described using perturbation theory - (Feynman diagrams) works well.
Quantum Chromodynamics asymptotic freedom: at high energies, coupling constant ‘goes’ to zero.

The nucleus, are seen as built of ‘free’ point-like particles= quarks.

\[ V(r) \]

“Strong Coulomb potential”
QCD: AT LOW ENERGIES
ASYMPTOTIC FREEDOM

\( \alpha_{QCD} > 1, \ V_{QCD}(r) \propto r \)

Non-perturbative confinement effect

No free quarks! they construct Hadrons:

Mesons (two quarks),
Baryons (three quarks),
...

Color Electric flux-tubes:
“a non-abelian Meissner effect”.

\[ V(r) \]
Static pot. for a pair of heavy quarks

Confinement

Coulomb
(some) OPEN PROBLEMS

– Phases of non-Abelian theories with fermionic matter
– Color superconductivity?
– Quark-gluon Plasma.
– Confinement/deconfinement of dynamical charges
– High-$T_c$ superconductivity?
LATTICE GAUGE THEORIES
We are all Wilsonians now

Posted on June 18, 2013 by preskill

Ken Wilson passed away on June 15 at age 77. He changed how we think about physics.

Renormalization theory, first formulated systematically by Freeman Dyson in 1949, cured the flaws of quantum electrodynamics and turned it into a precise computational tool. But the subject seemed magical and mysterious. Many physicists, Dirac prominently among them, questioned whether renormalization rests on a sound foundation.

Wilson changed that.

The renormalization group concept arose in an extraordinary paper by

Wilson also formulated the strong-coupling expansion of lattice gauge theory, and soon after pioneered the Euclidean Monte Carlo method for computing the quantitative non-perturbative predictions of quantum chromodynamics, which remains today an extremely active and successful program. But of the papers by Wilson I read while in graduate school, the most exciting by far was this one about the renormalization group. Toward the end of the paper Wilson discussed how to formulate the notion of the “continuum limit” of a field theory with a
Hamiltonian formulation of Wilson’s lattice gauge theories

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and Laboratory of Nuclear Studies, Cornell University, Ithaca, New York
(Received 9 July 1974)

Wilson’s lattice gauge model is presented as a canonical Hamiltonian theory. The structure of the model is reduced to the interactions of an infinite collection of coupled rigid rotators. The gauge-invariant configuration space consists of a collection of strings with quarks at their ends. The strings are lines of non-Abelian electric flux. In the strong-coupling limit the dynamics is best described in terms of these strings. Quark confinement is a result of the inability to break a string without producing a pair.
LATTICE GAUGE THEORY
LATTICE GAUGE THEORIES

HAMILTONIAN FORMULATION

Gauge group elements:

$U^r$ is an element of the gauge group (in the representation $r$), on each link

Left and right generators:

$$[L_a, U^r] = T_a^r U^r \quad ; \quad [R_a, U^r] = U^r T_a^r$$

$$[L_a, L_b] = -i f_{abc} L_c \quad ; \quad [R_a, R_b] = i f_{abc} R_c \quad ; \quad [L_a, R_b] = 0$$

$$\sum_a L_a L_a = \sum_a R_a R_a \equiv \sum_a E_a E_a$$

Gauge transformation:

$$U^r_{n,k} \rightarrow V^r_n U^r_{n,k} V^r_{n+k}$$

Generators:

$$(G_n)_a = \text{div}_n E_a = \sum_k \left( (L_{n,k})_a - (R_{n-k,k})_a \right)$$
LATTICE GAUGE THEORIES

HAMILTONIAN FORMULATION

Matter:

\[ \psi_n = (\psi_{n,a}) = \begin{pmatrix} \psi_{n,1} \\ \psi_{n,2} \\ \vdots \end{pmatrix} \]

Gauge transformation:

\[ \psi_n \rightarrow V_n^r \psi_n \]
Gauge field dynamics (Kogut-Susskind Hamiltonian):

\[
H_E = \frac{g^2}{2} \sum_{n,k,a} (E_{n,k})_a (E_{n,k})_a \\
H_B = -\frac{1}{g^2} \sum_{\text{plaquettes}} \left( \text{Tr} \left( U_1 U_2 U_3^\dagger U_4^\dagger \right) + h.c. \right)
\]

Strong coupling limit: \( g \gg 1 \)
Weak coupling limit: \( g \ll 1 \)

Matter dynamics:

\[
H_M = \sum_n M_n \psi_n^\dagger \psi_n \\
H_{int} = \varepsilon \sum_{n,k} (\psi_n^\dagger U_{n,k}^r \psi_{n+\hat{k}} + h.c.)
\]
TOY EXAMPLE: U(1)

\[ H = \sum_n M_n \psi_n^\dagger \psi_n + \epsilon \sum_n (\psi_n^\dagger \psi_{n+1} + h.c.) \]
TOY EXAMPLE: U(1)

\[ H = \sum_n M_n \psi_n^\dagger \psi_n + \epsilon \sum_n (\psi_n^\dagger \psi_{n+1} + h.c.) \]

H is invariant under global transformations:

\[ \psi_n \rightarrow e^{-i \Lambda} \psi_n \quad ; \quad \psi_n^\dagger \rightarrow \psi_n^\dagger e^{i \Lambda} \]
TOY EXAMPLE: $U(1)$

Promote the transformation to be **local**: 

$$\psi_n \longrightarrow e^{-i\Lambda_n} \psi_n \quad ; \quad \psi_n^\dagger \longrightarrow \psi_n^\dagger e^{i\Lambda_n}$$

Add a **new field** on the links:

$$U_n = e^{i\phi_n}$$
TOY EXAMPLE: U(1)

\[ H = \sum_n M_n \psi_n^\dagger \psi_n + \epsilon \sum_n (\psi_n^\dagger U_n \psi_{n+1} + h.c.) \]

Invariance under a **local** gauge transformations:

\[ \psi_n \rightarrow e^{-i\Lambda_n} \psi_n \quad ; \quad \psi_n^\dagger \rightarrow \psi_n^\dagger e^{i\Lambda_n} \]

\[ \phi_n \rightarrow \phi_n + \Lambda_{n+1} - \Lambda_n \]
TOY EXAMPLE: U(1)

Gauge field dynamics:

\[ H_E = \frac{g^2}{2} \sum_n L_n^2 \]

\[ L |m\rangle = m |m\rangle \]

\[ u_m(\phi) = \langle \phi | m \rangle = \frac{1}{\sqrt{2\pi}} e^{im\phi} \]

KS: “U(1) Rigid rotator”
Gauge field dynamics: PLAQUETTES

\[ H_B = -\frac{1}{2g^2} \sum_{\text{plaquettes}} U_1 U_2 U_3^\dagger U_4^\dagger + h.c. = \]

\[ -\frac{1}{g^2} \sum_{\text{plaquettes}} \cos (\phi_1 + \phi_2 - \phi_3 - \phi_4) \]

In the continuum limit, this REDUCES to \((\nabla \times A)^2\) : the magnetic energy density.
COMPACT QED (cQED)

\[ U_{n,k} = e^{i\phi_{n,k}} \]

\[ [E_{n,k}, \phi_{m,l}] = -i\delta_{nm}\delta_{kl} \]

Electric energy + Magnetic energy + Gauge-Matter interaction

\[
\frac{g^2}{2} \sum_{n,k} E_{n,k}^2 - \frac{1}{g^2} \sum_{n} \cos \left( \phi_{n,1} + \phi_{n+1,2} - \phi_{n+2,1} - \phi_{n,2} \right)
\]

\[
+ \epsilon \sum_{n,k} \left( \psi_n^\dagger e^{i\phi_{n,k}} \psi_{n+k} + \psi_{n+k}^\dagger e^{-i\phi_{n,k}} \psi_n \right)
\]

\[
+ M \sum_{n} (-1)^n \psi_n^\dagger \psi_n
\]
QUANTUM SIMULATIONS
Simulating Physics with Computers

Richard P. Feynman

Department of Physics, California Institute of Technology, Pasadena, California 91107

Received May 7, 1981

1. INTRODUCTION

On the program it says this is a keynote speech—and I don’t know what a keynote speech is. I do not intend in any way to suggest what should be in this meeting as a keynote of the subjects or anything like that. I have be understood very well in analyzing the situation. And I'm not happy with all the analyses that go with just the classical theory, because nature isn't classical, damnit, and if you want to make a simulation of nature, you'd better make it quantum mechanical, and by golly it's a wonderful problem, because it doesn't look so easy. Thank you.
REQUIREMENTS: HEP models

- **Fields**
  - Fermion Matter fields
  - Bosonic gauge fields

- **Local gauge invariance**
  - Exact, or low energy, effective

- **Relativistic invariance**
  - Causal structure, in the continuum limit
QUANTUM SIMULATION
COLD ATOMS

- Fermion matter fields
- Bosonic gauge fields

Superlattices:

\[ \psi_n = (\psi_{n,a}) = \begin{pmatrix} \psi_{n,1} \\ \psi_{n,2} \\ \vdots \end{pmatrix} \rightarrow \text{Atom internal levels} \]
1) EFFECTIVE GAUGE INVARIANCE

Gauss’s law is added as a constraint. Leaving the gauge invariant sector of Hilbert space costs too much Energy.

Low energy sector with a effective gauge invariant Hamiltonian.

\[ \Delta \gg \delta E \]

2) EXACT GAUGE INVARIANCE

- Atomic Symmetries \iff Local Gauge Invariance

**ABELIAN CASE:**

**NON-ABELIAN CASE:**
LINKS

\[ n + \hat{i} \]
\[ n \quad n + \hat{k} \]
cQED LINK

F-B Scattering
\( cQED \) LINK

\[ L_z \rightarrow L_z - 1 \]

\( \Phi_a, \Phi_b \)

\( \psi_c, \psi_d \)
cQED LINK

\( \Phi_a, \Phi_b \)

\( \psi_c \)

\( \psi_d \)
cQED LINK

Fermion

$\psi_c, \Phi_a, \Phi_b$

$\psi_d$

$L_z \rightarrow L_z + 1$
\[ \psi_c^\dagger \Phi_a^\dagger \Phi_b \psi_d + \psi_d^\dagger \Phi_b^\dagger \Phi_a \psi_c \]

\[ m_F (a) \]
\[ m_F (b) \]
\[ m_F (c) \]
\[ m_F (d) \]
\[ \psi_c \dagger \Phi_a \dagger \Phi_b \psi_d + \psi_d \dagger \Phi_b \dagger \Phi_a \psi_c \]

\[ m_F (a) \]

\[ m_F (b) \]

\[ m_F (c) \]

\[ m_F (d) \]
\[ \psi_c \dagger \Phi_a \dagger \Phi_b \psi_d + \psi_d \dagger \Phi_b \dagger \Phi_a \psi_c \]
\[ L_+ = \Phi_a \dagger \Phi_b \; ; \; L_- = \Phi_b \dagger \Phi_a \]
\[ L_z = \frac{1}{2} (N_a - N_b) \; ; \; l = \frac{1}{2} (N_a + N_b) \]
and thus what we have is

\[ L_+ = \Phi_a^\dagger \Phi_b \quad ; \quad L_- = \Phi_b^\dagger \Phi_a \]

\[ L_z = \frac{1}{2} (N_a - N_b) \quad ; \quad l = \frac{1}{2} (N_a + N_b) \]

and thus what we have is

\[ \psi_c^\dagger \Phi_a^\dagger \Phi_b \psi_d + \psi_d^\dagger \Phi_b^\dagger \Phi_a \psi_c \]

\[ \psi_c^\dagger L_+ \psi_d \sim \psi_c^\dagger e^{i\theta} \psi_d \]

where for large \( l \), \( m \ll l \)

\[ L_+ \sim e^{i(\phi_1 - \phi_2)} \equiv e^{i\theta} \]
DYNAMICAL FERMIONS

Staggered Fermions"
DYNAMICAL FERMIONS
SHWINGER MODEL

\[ \frac{\epsilon}{\sqrt{\ell (\ell + 1)}} \sum_n \left( \psi_n^\dagger L_{+,n} \psi_{n+1} + h.c. \right) \]
NON-ABELIAN LINK

\[ \psi_n = (\psi_{n,a}) = \begin{pmatrix} \psi_{n,1} \\ \psi_{n,2} \\ \vdots \end{pmatrix} \]

\[ U^r = \text{element of the gauge group} \]

\[ H_{int} = \epsilon \sum_{n,k} (\psi_n^\dagger U^r_{n,k} \psi_{n+k} + h.c.) \]
NON-ABELIAN LINKS

\[ H_{\text{link}} = \{ \left| j, m, m' \right> \} = \bigoplus_j \left[ j_L \otimes j_R \right]_{\text{max}E} \]

\[
\begin{align*}
[L_a, R_b] &= 0 \\
[L_a, U^r] &= T_a^r U^r \\
[R_a, U^r] &= U^r T_a^r \\
[L_a, L_b] &= -if_{abc} L_c \\
[R_a, R_b] &= if_{abc} R_c \\
\sum_a L_a L_a &= \sum_a R_a R_a = \sum_a E_a E_a
\end{align*}
\]
SU(2) EXACT

\[ H_{link} = 0 \oplus \left( \frac{1}{2} \otimes \frac{1}{2} \right) \]
SU(2) EFFECTIVE

Ancillary “constraint” Fermion

“color” fermions

\[ U_L = \frac{1}{\sqrt{N_L + 1}} \begin{pmatrix} a_1^\dagger & -a_2 \\ a_2 & a_1 \end{pmatrix} ; \quad U_R = \begin{pmatrix} b_1^\dagger & b_2^\dagger \\ -b_2 & b_1 \end{pmatrix} \frac{1}{\sqrt{N_R + 1}} \]

\[ U = U_L U_R \]

On each link – \( a_{1,2} \) bosons on the left, \( b_{1,2} \) bosons on the right
PLAQUETTES
1d elementary link interactions – already gauge invariant building blocks of effective plaquettes

Auxiliary fermions := ♦️
1d elementary link interactions – *already gauge invariant* building blocks of effective plaquettes

Auxiliary fermions := ⬤
1d elementary link interactions – already gauge invariant building blocks of effective plaquettes

Auxiliary fermions
– virtual processes
1d elementary link interactions – already gauge invariant building blocks of effective plaquettes

Auxiliary fermions
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1d elementary link interactions – already gauge invariant building blocks of effective plaquettes

Auxiliary fermions
– virtual processes
- plaquettes.

\[
\sum_{\text{plaquettes}} \left( \text{Tr} \left(U_1 U_2 U_3^\dagger U_4^\dagger\right) + h.c. \right)
\]
1d elementary link interactions – already gauge invariant building blocks of effective plaquettes

Auxiliary fermions
– virtual processes
- plaquettes.

\[ \sum_{\text{plaquettes}} \left( \text{Tr} \left( U_1 U_2 U_3^\dagger U_4^\dagger \right) + h.c. \right) \]

OKAY for: discrete, abelian & non-abelian groups
Three atomic layers w. movable control atoms

Lattice Gauge Theory with Stators

Matter Fermions
Link (Gauge) degrees of freedom
Control degrees of freedom

Digital Lattice Gauge Theories

The $Z_2$ example:

- **Plaquette interactions**
  \[ \sigma_x(x, 1) \sigma_x(x + \hat{1}, 2) \sigma_x(x + \hat{2}, 1) \sigma_x(x, 2) \]

- **Link interactions**
  \[ \psi^\dagger(x) \sigma_x(x, k) \psi(x + \hat{k}) \]
Plaquettes: Four-body Interactions

Stators: two-body interactions $\rightarrow$ four-body interactions

$|\tilde{i_n}\rangle = \frac{1}{\sqrt{2}} \left( |\uparrow\rangle + |\downarrow\rangle \right)$

$\mathcal{U}_4 |\tilde{i_n}\rangle = \frac{1}{\sqrt{2}} \left( |\uparrow\rangle + \sigma_4^x \otimes |\downarrow\rangle \right)$
Plaquettes: Four-body Interactions

Stators: two-body interactions $\rightarrow$ four-body interactions

$$\left| \tilde{\text{i}}\text{n} \right\rangle = \frac{1}{\sqrt{2}} \left( \left| \uparrow \right\rangle + \left| \downarrow \right\rangle \right)$$

$$u_4^\dagger \left| \tilde{\text{i}}\text{n} \right\rangle = \frac{1}{\sqrt{2}} \left( \left| \uparrow \right\rangle + \sigma_4^x \otimes \left| \downarrow \right\rangle \right)$$

$$u_3^\dagger u_4^\dagger \left| \tilde{\text{i}}\text{n} \right\rangle = \frac{1}{\sqrt{2}} \left( \left| \uparrow \right\rangle + \sigma_3^x \sigma_4^x \otimes \left| \downarrow \right\rangle \right)$$
Plaquettes: Four-body Interactions

Stators: two-body interactions $\rightarrow$ four-body interactions

$|\tilde{in}\rangle = \frac{1}{\sqrt{2}} \left( |\uparrow\rangle + |\downarrow\rangle \right)$

$U_4^\dagger |\tilde{in}\rangle = \frac{1}{\sqrt{2}} \left( |\uparrow\rangle + \sigma_4^x \otimes |\downarrow\rangle \right)$

$U_3^\dagger U_4^\dagger |\tilde{in}\rangle = \frac{1}{\sqrt{2}} \left( |\uparrow\rangle + \sigma_3^x \sigma_4^x \otimes |\downarrow\rangle \right)$

$U_2^\dagger U_3^\dagger U_4^\dagger |\tilde{in}\rangle = \frac{1}{\sqrt{2}} \left( |\uparrow\rangle + \sigma_2^x \sigma_3^x \sigma_4^x \otimes |\downarrow\rangle \right)$
Plaquettes: Four-body Interactions

Stators: two-body interactions $\rightarrow$ four-body interactions

$$|\tilde{in}\rangle = \frac{1}{\sqrt{2}} \left( |\uparrow\rangle + |\downarrow\rangle \right)$$

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$$U_2^\dagger U_3^\dagger U_4^\dagger |\tilde{in}\rangle = \frac{1}{\sqrt{2}} \left( |\uparrow\rangle + \sigma_2^x \sigma_3^x \sigma_4^x \otimes |\downarrow\rangle \right)$$

$$U_1^\dagger U_2^\dagger U_3^\dagger U_4^\dagger |\tilde{in}\rangle = \frac{1}{\sqrt{2}} \left( |\uparrow\rangle + \sigma_1^x \sigma_2^x \sigma_3^x \sigma_4^x \otimes |\downarrow\rangle \right)$$
Plaquettes: Four-body Interactions

Stators: two-body interactions \(\rightarrow\) four-body interactions

\[
|\tilde{in}\rangle = \frac{1}{\sqrt{2}} \left( |\uparrow\rangle + |\downarrow\rangle \right)
\]

\[
U_4^\dagger |\tilde{in}\rangle = \frac{1}{\sqrt{2}} \left( |\uparrow\rangle + \sigma_4^x \otimes |\downarrow\rangle \right)
\]

\[
U_3^\dagger U_4^\dagger |\tilde{in}\rangle = \frac{1}{\sqrt{2}} \left( |\uparrow\rangle + \sigma_3^x \sigma_4^x \otimes |\downarrow\rangle \right)
\]

\[
U_2 U_3^\dagger U_4^\dagger |\tilde{in}\rangle = \frac{1}{\sqrt{2}} \left( |\uparrow\rangle + \sigma_2^x \sigma_3^x \sigma_4^x \otimes |\downarrow\rangle \right)
\]

\[
U_1 U_2 U_3^\dagger U_4^\dagger |\tilde{in}\rangle = \frac{1}{\sqrt{2}} \left( |\uparrow\rangle + \sigma_1^x \sigma_2^x \sigma_3^x \sigma_4^x \otimes |\downarrow\rangle \right)
\]

\[
S = \frac{1}{\sqrt{2}} \left( |\uparrow\rangle + \sigma_4^x \otimes |\downarrow\rangle \right)
\]
Plaquettes: Four-body Interactions

Stators: two-body interactions $\rightarrow$ four-body interactions

$S_\square = \frac{1}{\sqrt{2}} \left( |\uparrow\rangle + \sigma_\square^x \otimes |\downarrow\rangle \right)$

$\tilde{\sigma}^x S_\square = S_\square \sigma_\square^x$

$e^{-i \lambda \tilde{\sigma}^x \tau} S_\square = S_\square e^{-i \lambda \sigma_\square^x \tau}$

$U_4 U_3 U_2 U_1 e^{-i \lambda \tilde{\sigma}^x \tau} U_1 U_2 U_3 U_4 |\tilde{\text{in}}\rangle = |\tilde{\text{in}}\rangle e^{-i \lambda \sigma_\square^x \tau}$
A bipartite single time step Trotterized time evolution, of already gauge invariants elements

\[ e^{-i\sum_j H_j t} = \lim_{M \to \infty} \left( \prod_j e^{-i H_j \frac{t}{M}} \right)^M \]

\[ \left\| \left( \tilde{\mathcal{L}}_M^{(2)}(t) - \mathcal{L}(t) \right) \right\| \leq 60 \frac{t^3 L^6 \lambda_{\text{max}}^3}{M^2} . \]
Experimental Demonstration of Single-Site Addressability in a Two-Dimensional Optical Lattice

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(Received 18 March 2009; published 21 August 2009)

FIG. 1 (color online). Electron microscope image of a Bose-Einstein condensate in a 2D optical lattice with 600 nm lattice spacing (sum obtained from 260 individual experimental realizations). Each site has a tubelike shape with an extension of 6 μm perpendicular to the plane of projection. The central lattice sites contain about 80 atoms.

FIG. 2 (color online). Patterning a Bose-Einstein condensate in a 2D optical lattice with a spacing of 600 nm. Every emptied site was illuminated with the electron beam (7 nA beam current, 100 nm FWHM beam diameter) for (a),(b) 3, (c),(d) 2, and (e) 1.5 ms, respectively. The imaging time was 45 ms. Between 150 and 250 images from individual experimental realizations have been summed for each pattern.
A quantum gas microscope for detecting single atoms in a Hubbard-regime optical lattice

Waseem S. Bakr¹, Jonathon I. Gillen¹, Amy Peng¹, Simon Fölling¹ & Markus Greiner¹

mined through preparation and measurement. By implementing a high-resolution optical imaging system, single atoms are detected with near-unity fidelity on individual sites of a Hubbard-regime optical lattice. The lattice itself is generated by projecting a holographic mask through the imaging system. It has an arbitrary geometry, chosen to support both strong tunnel coupling between lattice sites and strong on-site confinement. Our approach can be

Figure 3 | Site-resolved imaging of single atoms on a 640-nm-period optical lattice, loaded with a high density Bose–Einstein condensate. Inset, magnified view of the central section of the picture. The lattice structure and the discrete atoms are clearly visible. Owing to light-assisted collisions and molecule formation on multiply occupied sites during imaging, only empty and singly occupied sites can be seen in the image.
Single-spin addressing in an atomic Mott insulator

Christof Weitenberg, Manuel Endres, Jacob F. Sherson, Marc Cheneau, Peter Schauß, Takeshi Fukuhara, Immanuel Bloch & Stefan Kuhr

Figure 2 | Single-site addressing. a, Top, experimentally obtained fluorescence image of a Mott insulator with unity filling in which the spin of selected atoms was flipped from \( \langle 0 \rangle \) to \( \langle 1 \rangle \) using our single-site addressing scheme. Atoms in state \( \langle 1 \rangle \) were removed by a resonant laser pulse before detection. Bottom, the reconstructed atom number distribution on the lattice. Each filled circle indicates a single atom; the points mark the lattice sites. b, Top, as for a except that a global microwave sweep exchanged the population in \( \langle 0 \rangle \) and \( \langle 1 \rangle \), such that only the addressed atoms were observed. Bottom, the reconstructed atom number distribution shows 14 atoms on neighbouring sites. c–f, As for b, but omitting the atom number distribution. The images contain 29 (c), 35 (d), 18 (e) and 23 (f) atoms. The single isolated atoms in b, e and f were placed intentionally to allow for the correct determination of the lattice phase for the feedback on the addressing beam position.
Real-time dynamics of lattice gauge theories with a few-qubit quantum computer

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systems. In contrast, quantum simulations aim at the long-term goal of solving the specific yet fundamental class of problems that currently cannot be tackled by these classical techniques. The digital approach we employ here is based on the Hamiltonian formulation of gauge theories9, and enables direct access to the system wavefunction. As we show below, this allows us to investigate entanglement generation during particle–antiparticle production, emphasizing a novel perspective on the dynamics of the Schwinger mechanism2.
Implementing quantum electrodynamics with ultracold atomic systems

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Oberthaler group
4. Microscopic parameters

At this point, we are now able to determine the accessible parameters for an experimental implementation of the Schwinger model via a mixture of bosonic $^{23}$Na and fermionic $^6$Li atoms [30], which is determined by the parameters $\chi_{BB}$, $\chi_{BF}$, $\Lambda$ and the occupation numbers of the links.
CONFINEMENT
TOY MODELS

• 1+1D: Schwinger’s model.

• cQED: 2+1D: no phase transition
  Instantons give rise to confinement at $g < 1$ (Polyakov).
  (For $T > 0$: there is a phase transition also in 2+1D.)

• cQED: 3+1D: phase transition between a strong coupling confining phase, and a weak coupling coulomb phase.

• $Z(N)$: for $N \geq N_c$: Three phases: electric confinement, magnetic confinement, and non confinement.
References


Hep simulations/GROUPS

– ICFO, Barcelona (Lewenstein)
– Innsbruck (Zoller, Blatt)
– University of Bern (Wiese)
– Heidelberg (Oberthaler, Berges)
– UGENT (Verstraete)
– Caltech, UMD (Preskill, Jordan)
– …
– Tensor Networks with LGI (cQED in 1+1, 2+1), MPQ, UGENT, Ulm, Mainz
Hep simulations/GROUPS

– ICFO, Barcelona (Lewenstein)
– Innsbruck (Zoller, Blatt)
– University of Bern (Wiese)
– Heidelberg (Oberthaler, Berges)
– UGENT (Verstraete)
– ...

Thank You!