Quantum Fields, Gravity, and Complexity

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JOINT CENTER FOR QUANTUM INFORMATION AND COMPUTER SCIENCE

It from Qubit
Simons Collaboration on Quantum Fields, Gravity and Information

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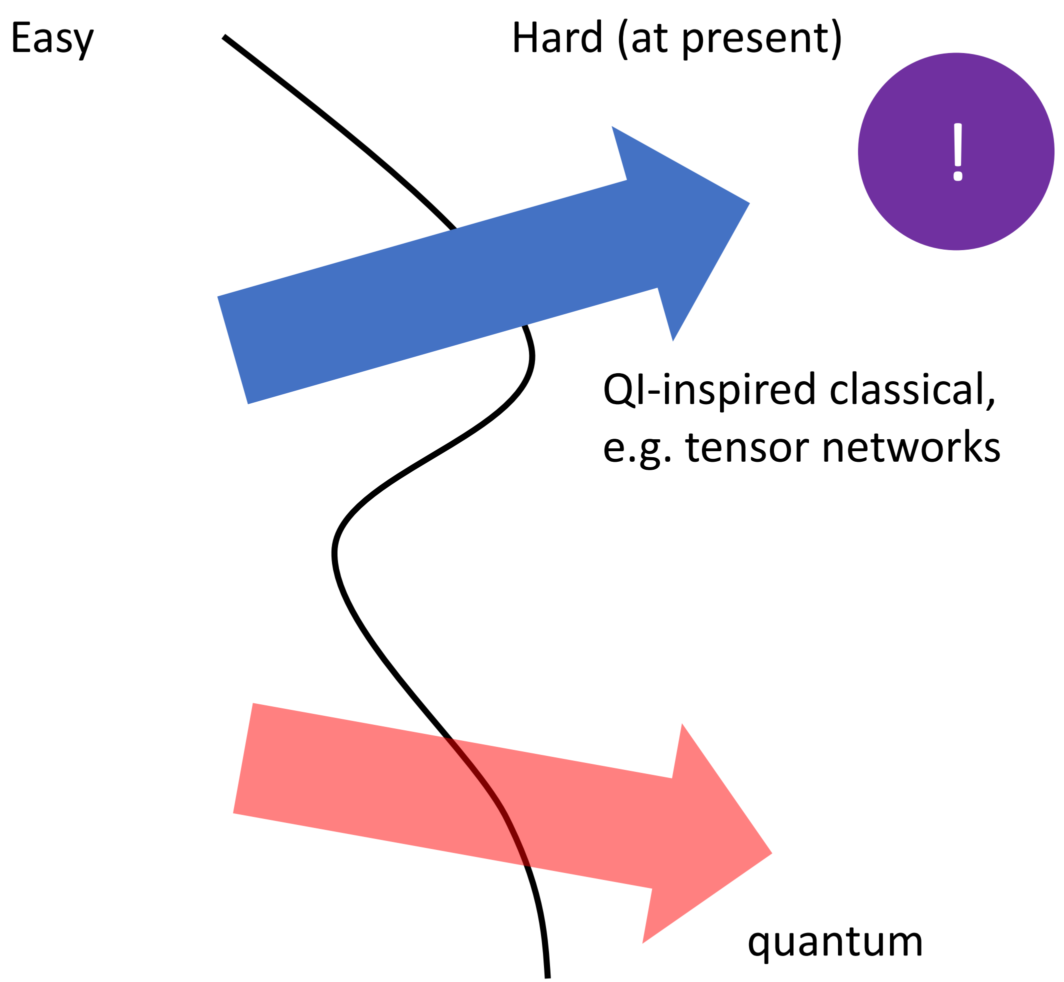
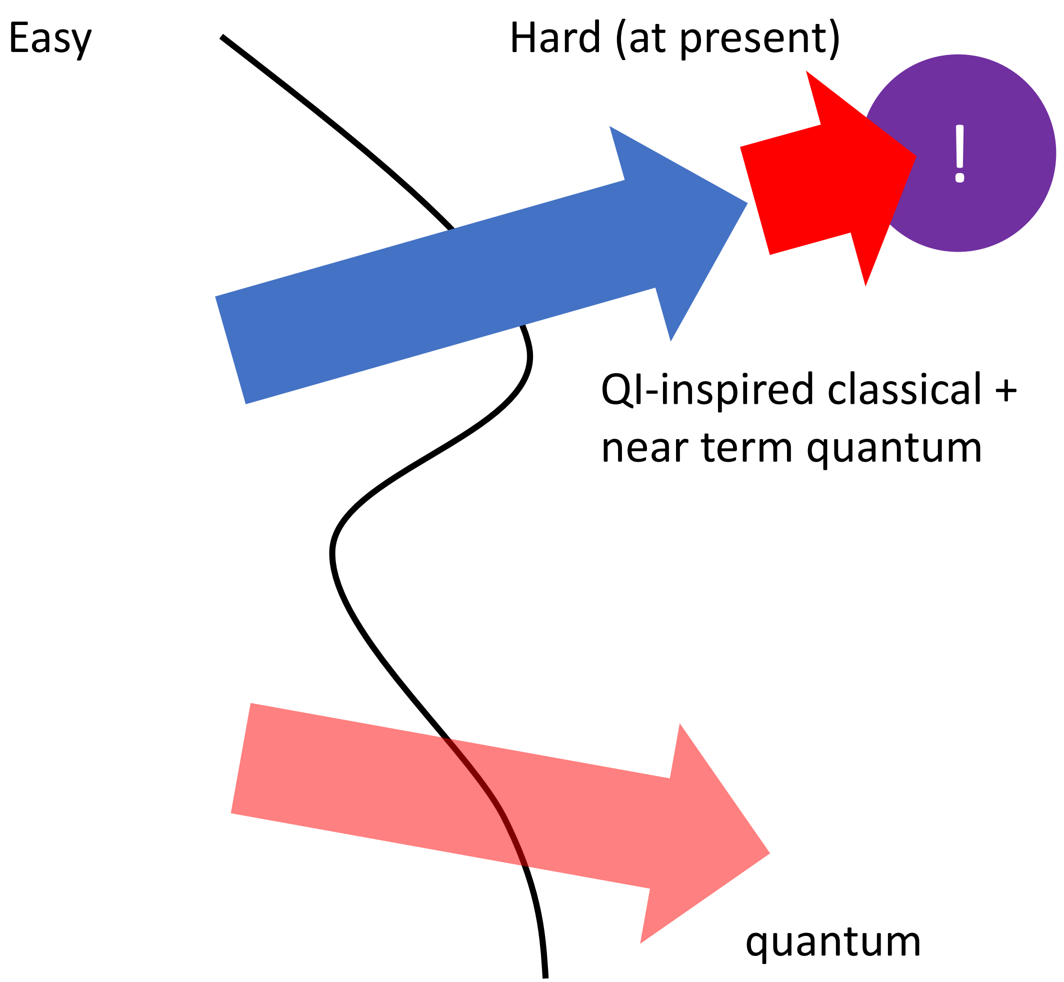
Influence of quantum information





Quantity and quality of quantum resources needed to simulate an interesting quantum field theory?

Quantity and quality of quantum resources needed to simulate an interesting quantum gravity?

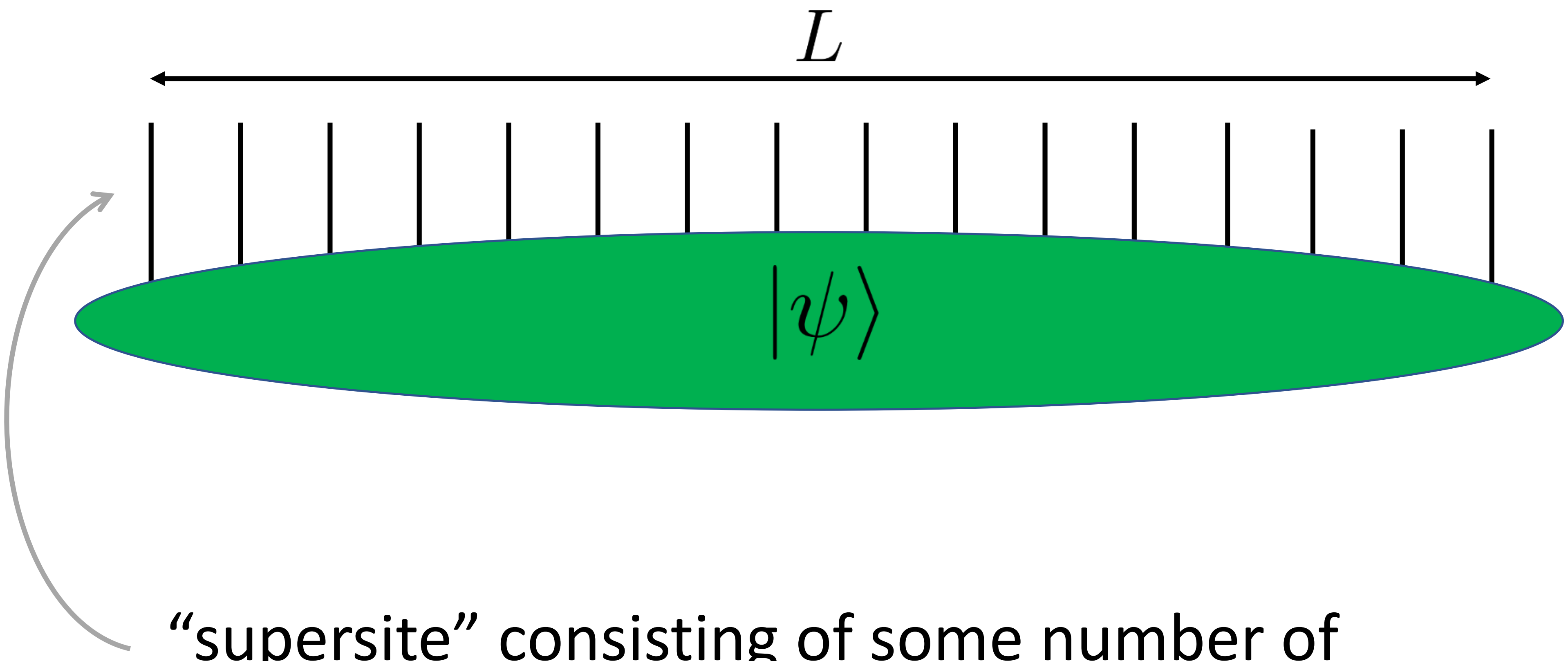
Clearly, the most favorable answer will utilize as much "classical precomputing" as possible

What to simulate? How to simulate?

- What: Simplest focus is probably ground state physics of local Hamiltonians, especially few-body correlation functions, local properties, thermodynamic data (and generalizations to finite temperature)

- How: Tensor networks (based on entanglement structure) contracted either classically or using smallish quantum devices → "Quantum Assisted Classical Simulation"

[Calder SFMOMA]

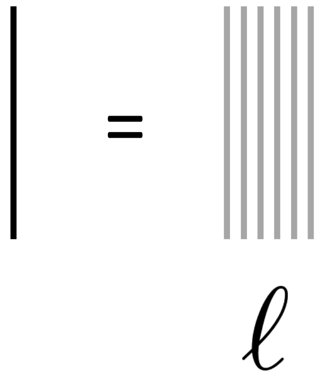


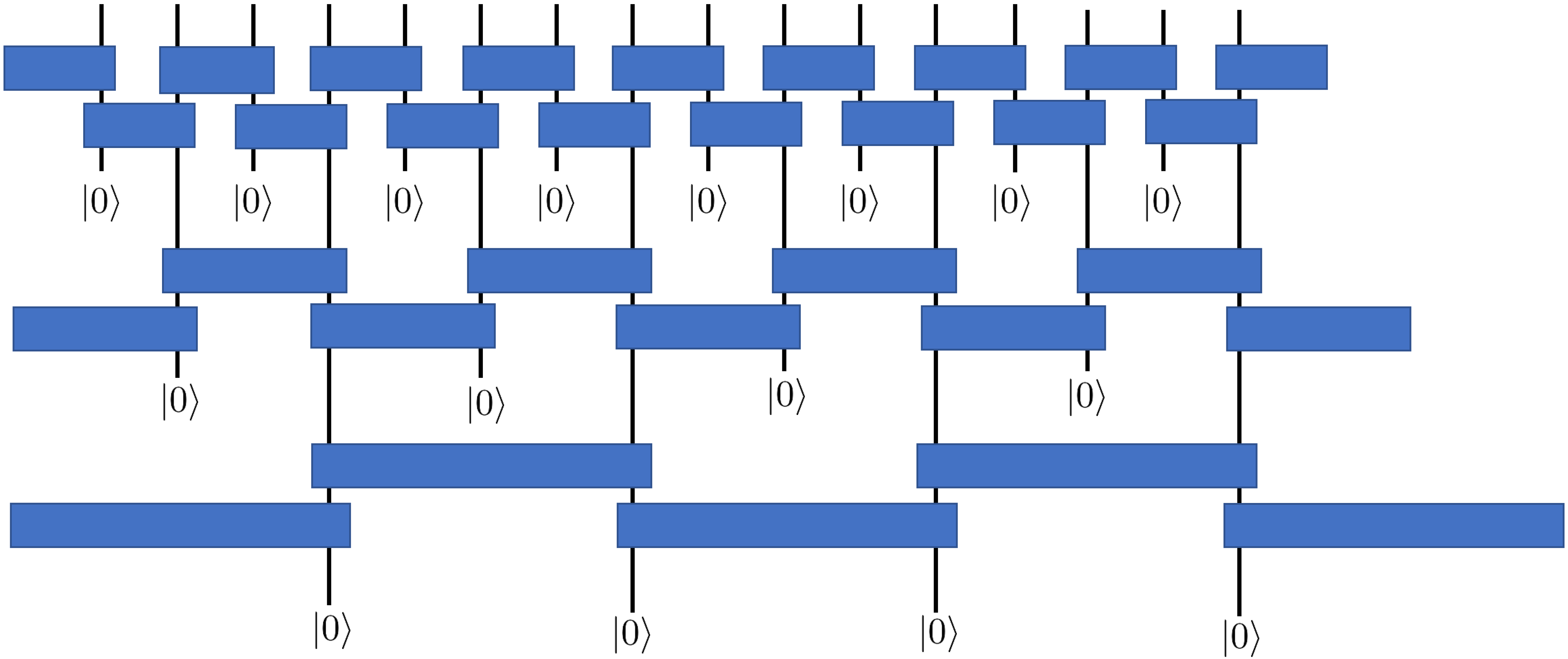
"supersite" consisting of some number of fine-grained degrees of freedom

$$\dim(\mathcal{V}) = \chi$$

or

$\ell = \log_2 \chi$ qubits





+ generalizations to higher dimensions

[MERA: Vidal]

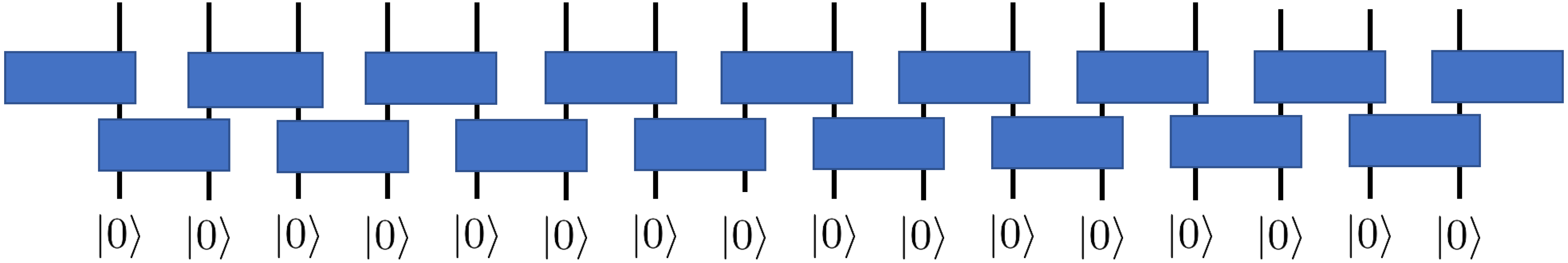
Characteristic scales

- Lattice spacing a
- Correlation length ξ
- Entanglement range ξ_E
- Supersite size ℓ

“Trivial” gapped states (lattice scale)

$$\xi \sim \xi_E \sim a \qquad \ell \sim \xi$$

- Examples: non-topological band insulators like diamond, mean-field superconductors, …
- Physics: There exists a gapped path in Hamiltonian space, H(s), such that H(0) has a trivial ground state and H(1) has a product ground state → use adiabatic preparation (Andrew’s talk)
- Low energy field theory: n/a



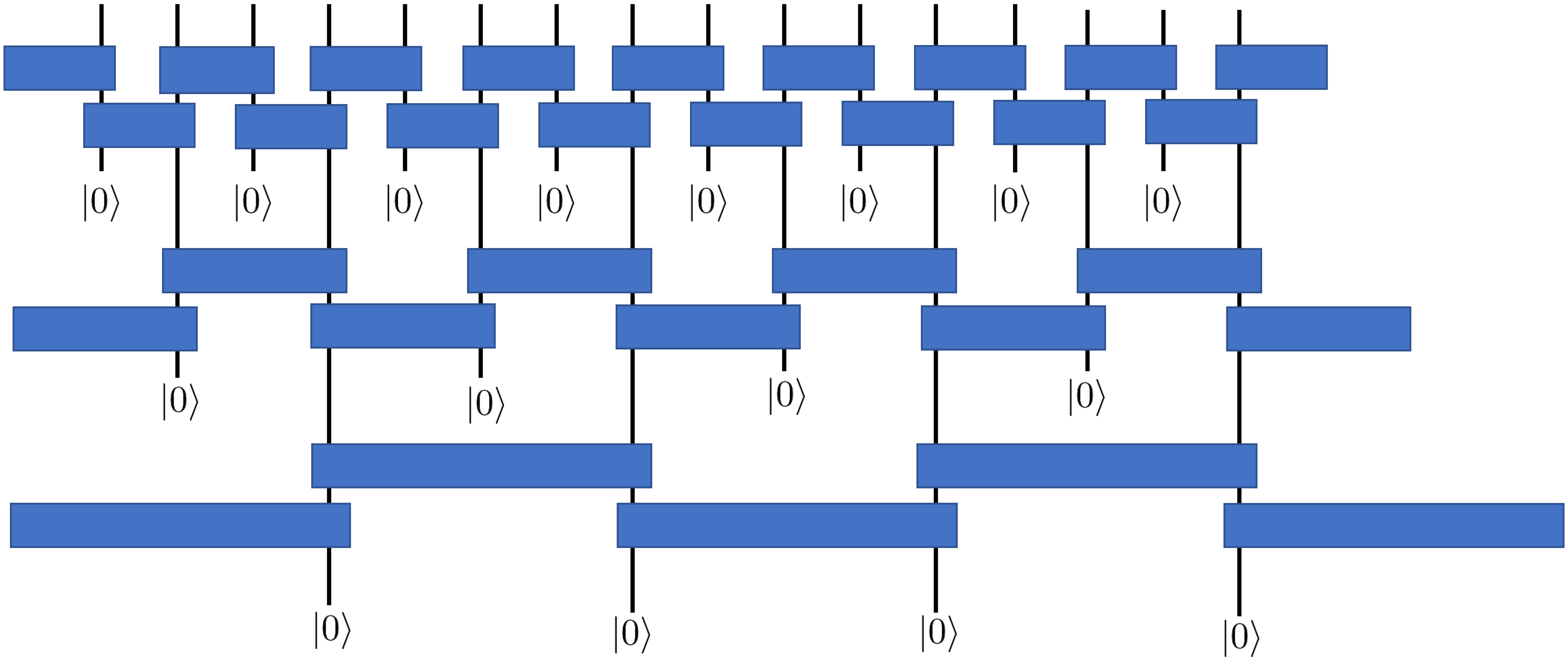
+ generalizations to higher dimensions

"Topological" gapped states (lattice scale)

$$\xi \sim a \qquad \ell \sim \xi \qquad \xi_E \to \infty$$

- Examples: quantum Hall states, discrete gauge theory (toric code), some spin liquids, …
- Physics: There exists a gapped path in Hamiltonian space, H(s), such that H(0) is size L and H(1) is size 2L → a kind of scale invariance
- Low energy field theory: topological

[S-McGreevy 1407.8203]



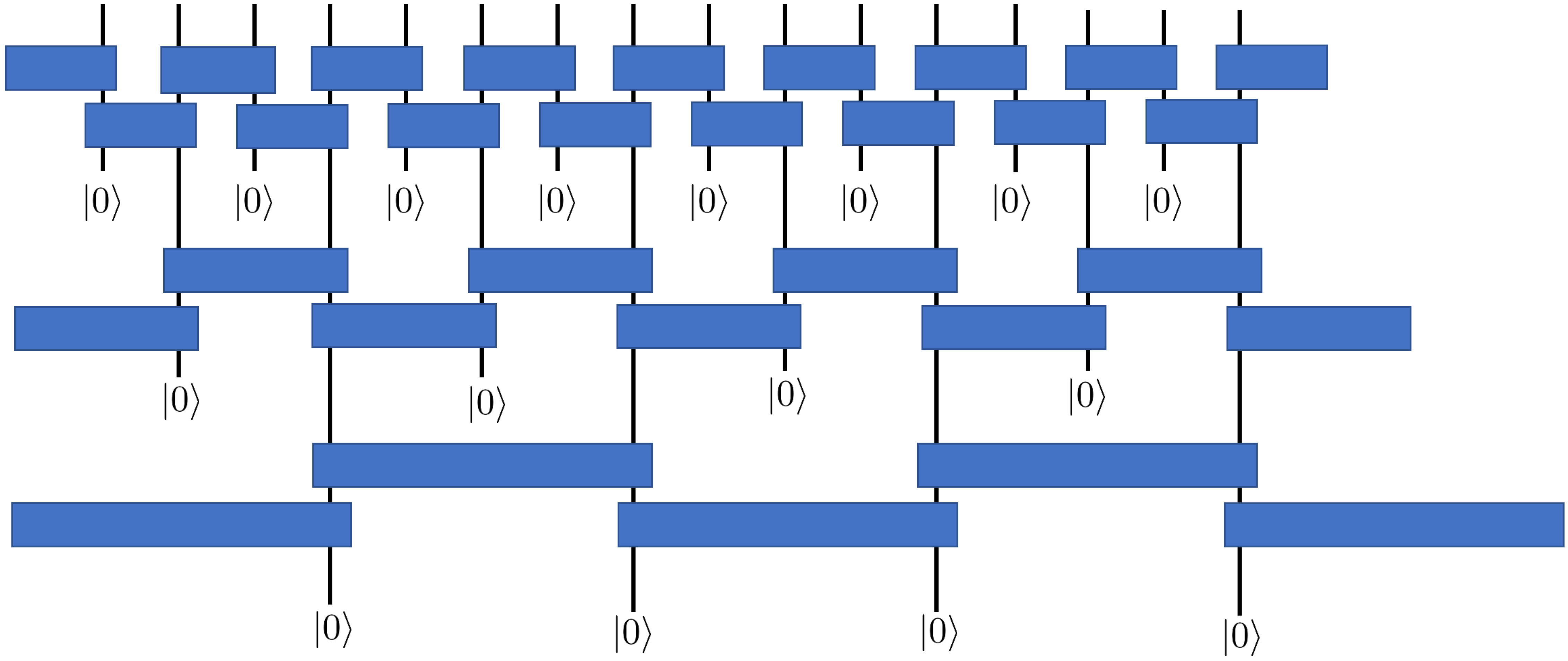
+ generalizations to higher dimensions

Gapless states

$$\ell \sim a \qquad \xi \to \infty \qquad \xi_E \to \infty$$

- Examples: non-interacting fermions, quantum critical points, …
- Physics: fixed point of renormalization group implies some kind of equivalence between size L and size 2L; no general argument, but various approaches including wavelets (Jutho's talk), etc.
- Low energy field theory: conformal field theory, scale invariant field theory (non-Lorentz invariant)

[wavelets: Evenbly-White, Haegeman et al.]

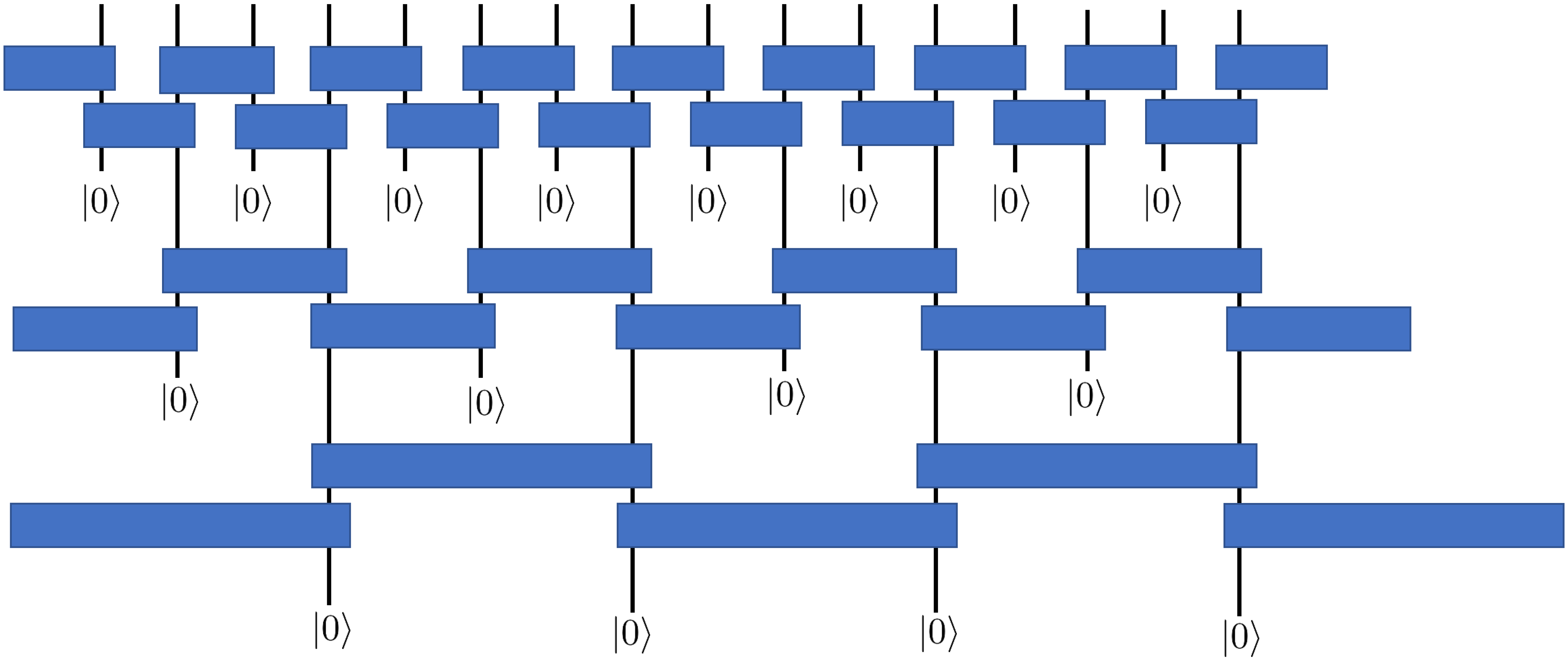


+ generalizations to higher dimensions

Holographic states

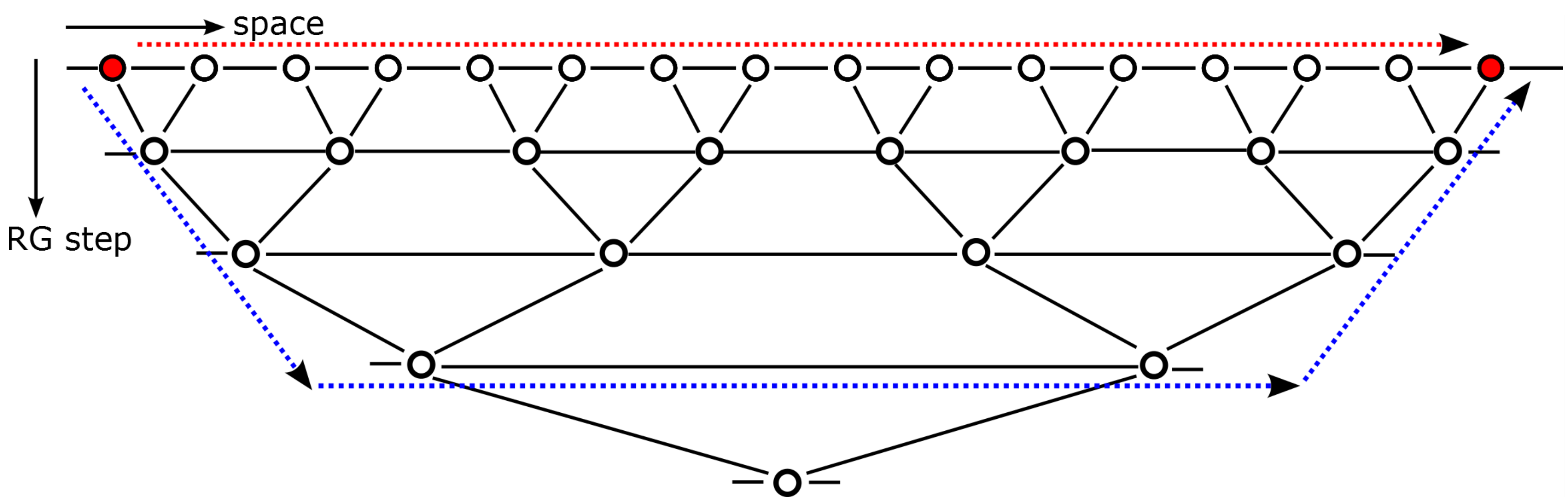
$$\ell \sim \ell_0 \gg 1 \qquad \xi \to \infty \qquad \xi_E \to \infty$$

- Examples: lattice regulated N=4 SYM, ….
- Physics: fixed point of renormalization group implies some kind of equivalence between size L and size 2L; entanglement as the "fabric" of dual holographic spacetime [S 0905.1317, Van Raamsdonk, …]
- Low energy field theory: conformal field theory, bulk gravitational dual



+ generalizations to higher dimensions

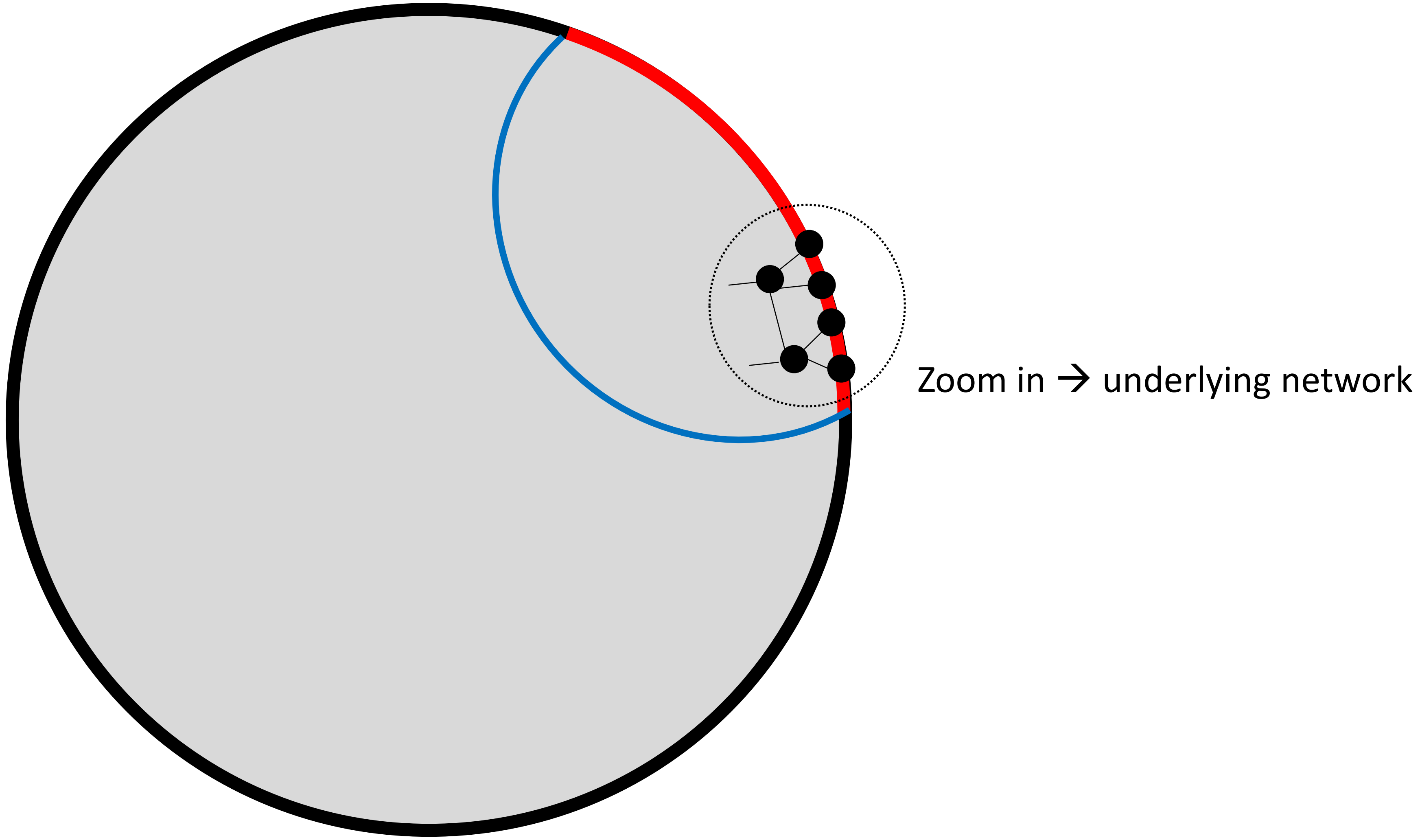
Discrete hyperbolic geometry

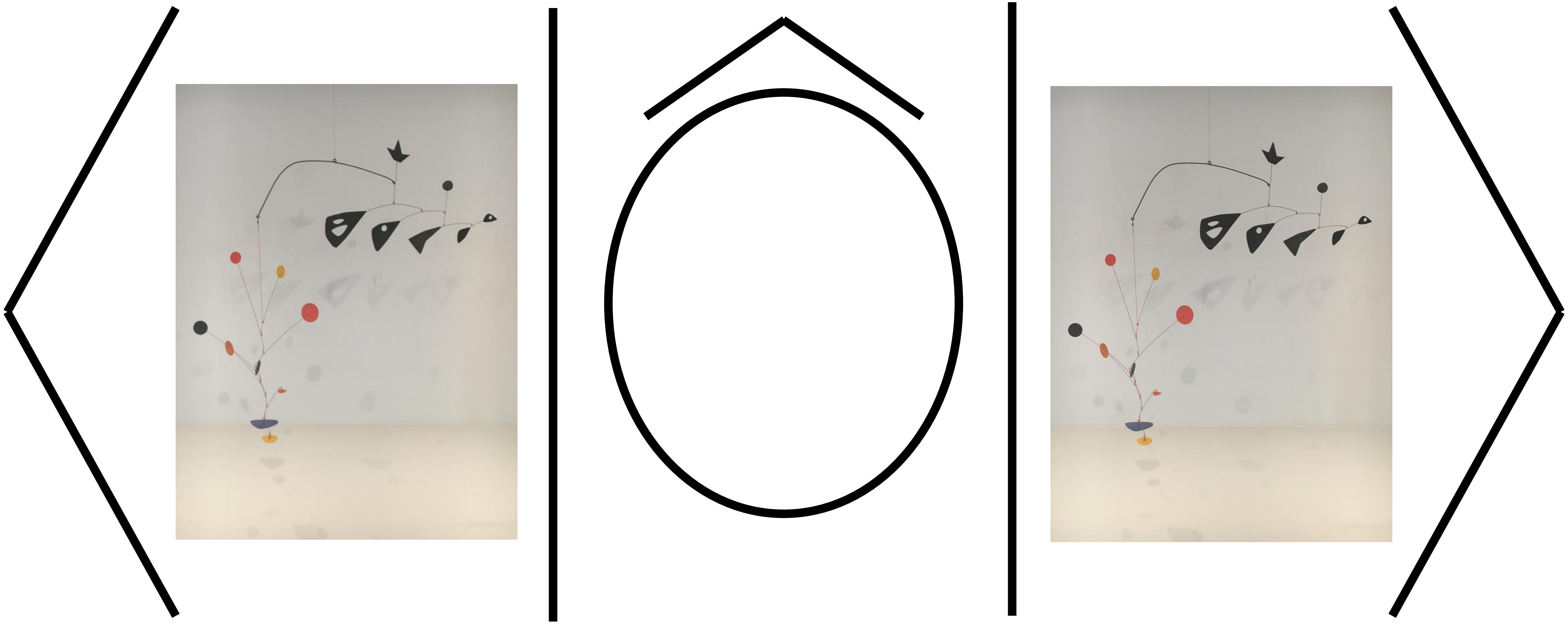


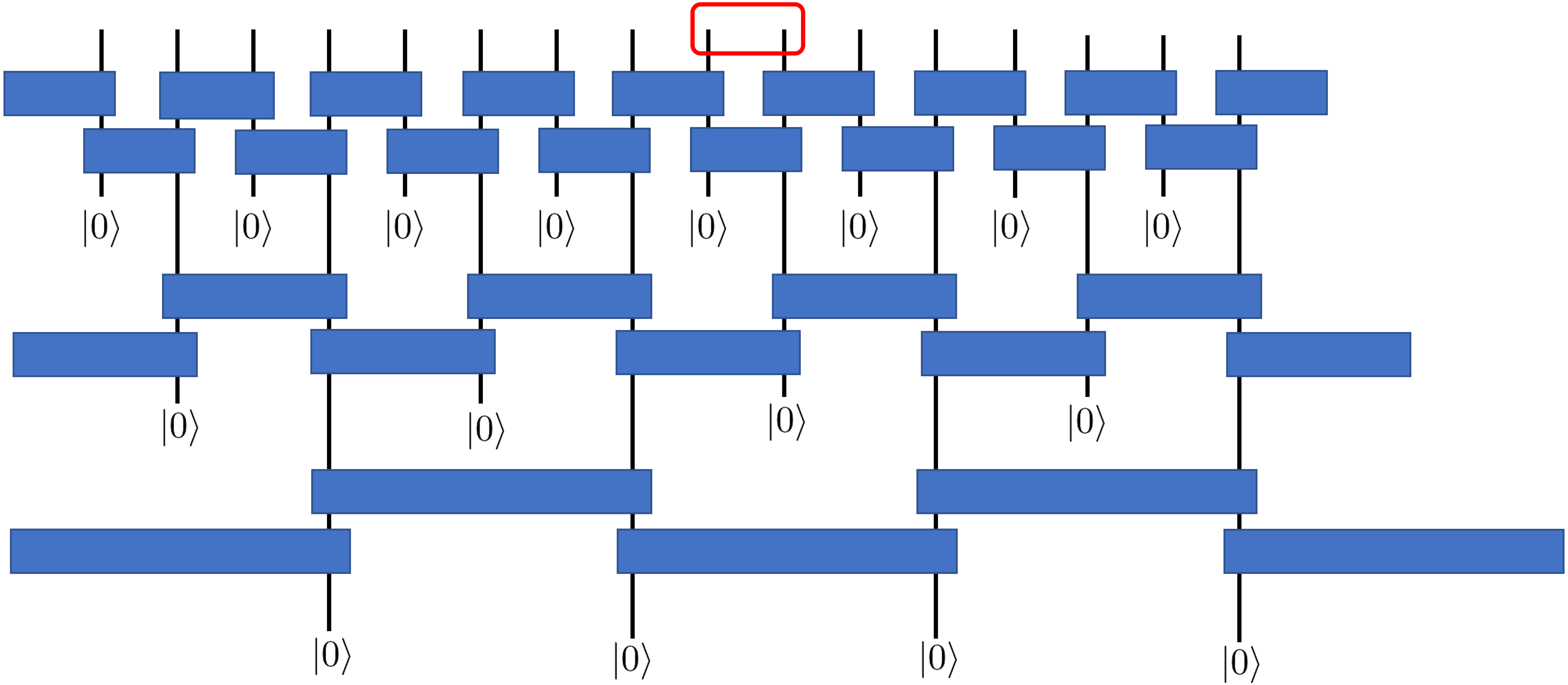
Entanglement obeys an area law, but in an emergent geometry! [S 0905.1317, Ryu-Takayanagi]

Mimics AdS: $$ds^2 = \frac{\ell^2}{r^2}(dr^2 + dx^2)$$

AdS/CFT:



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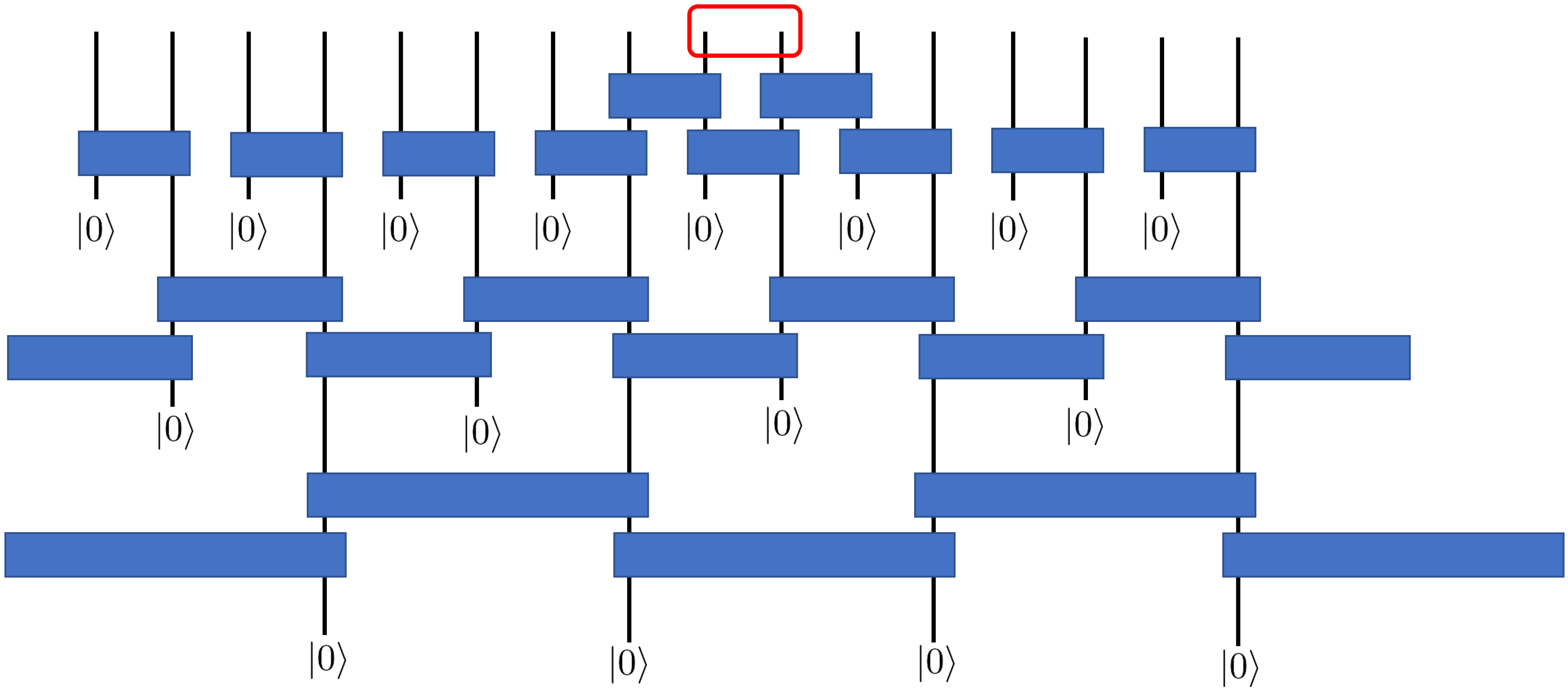
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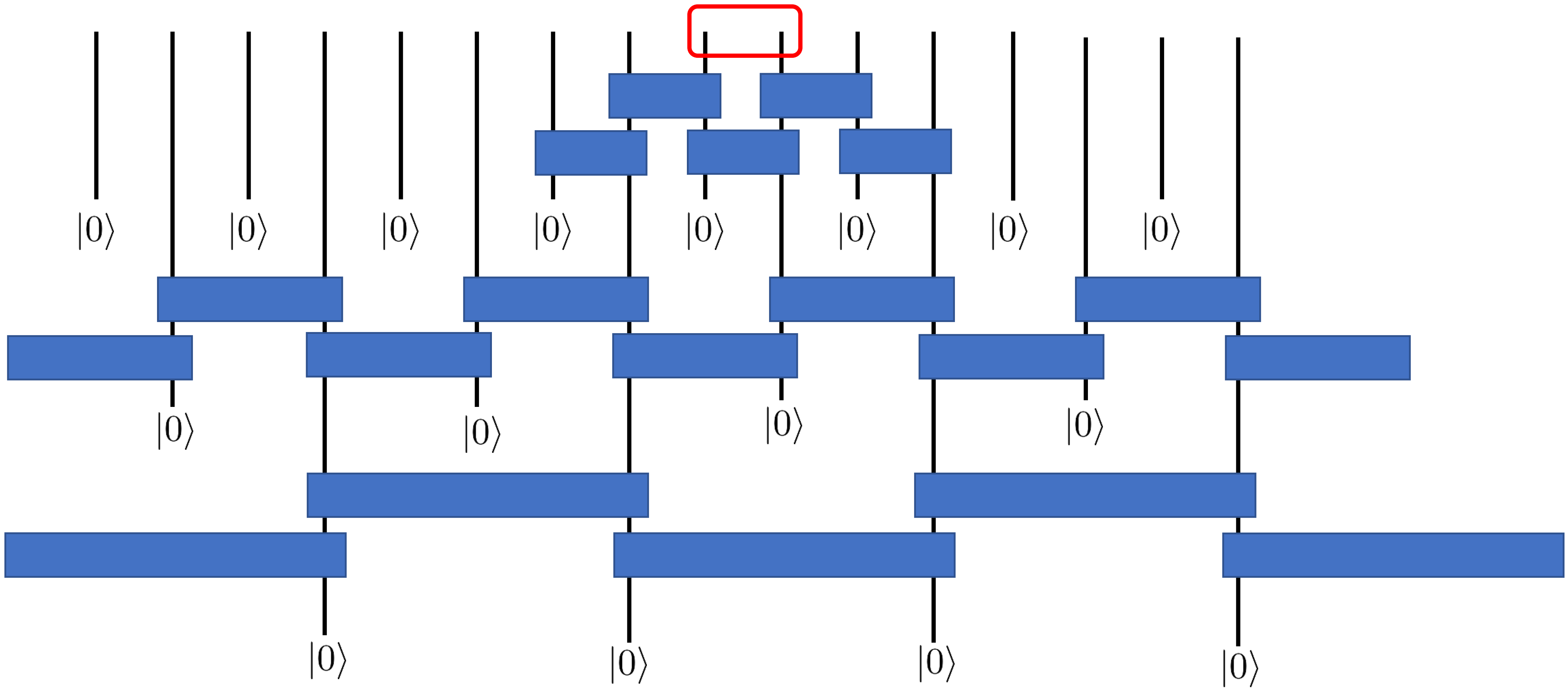

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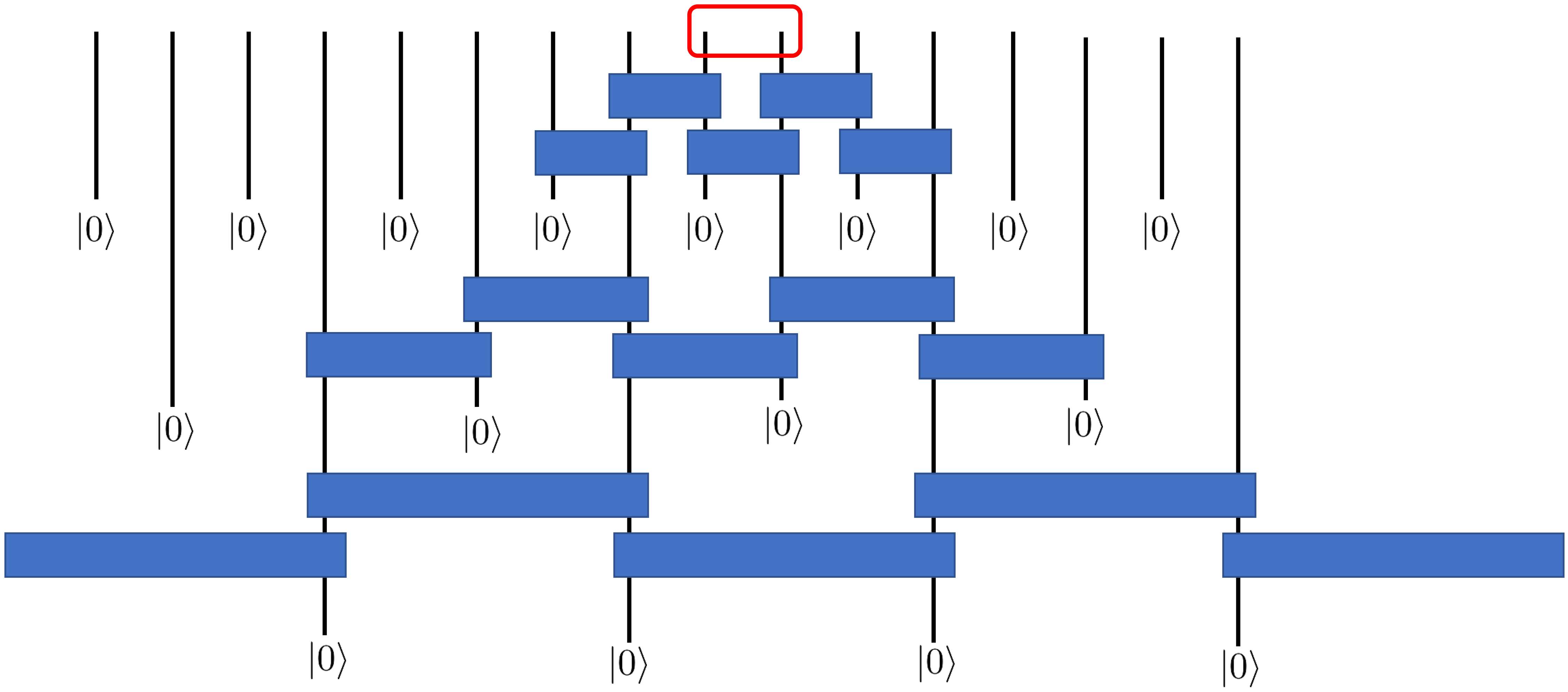
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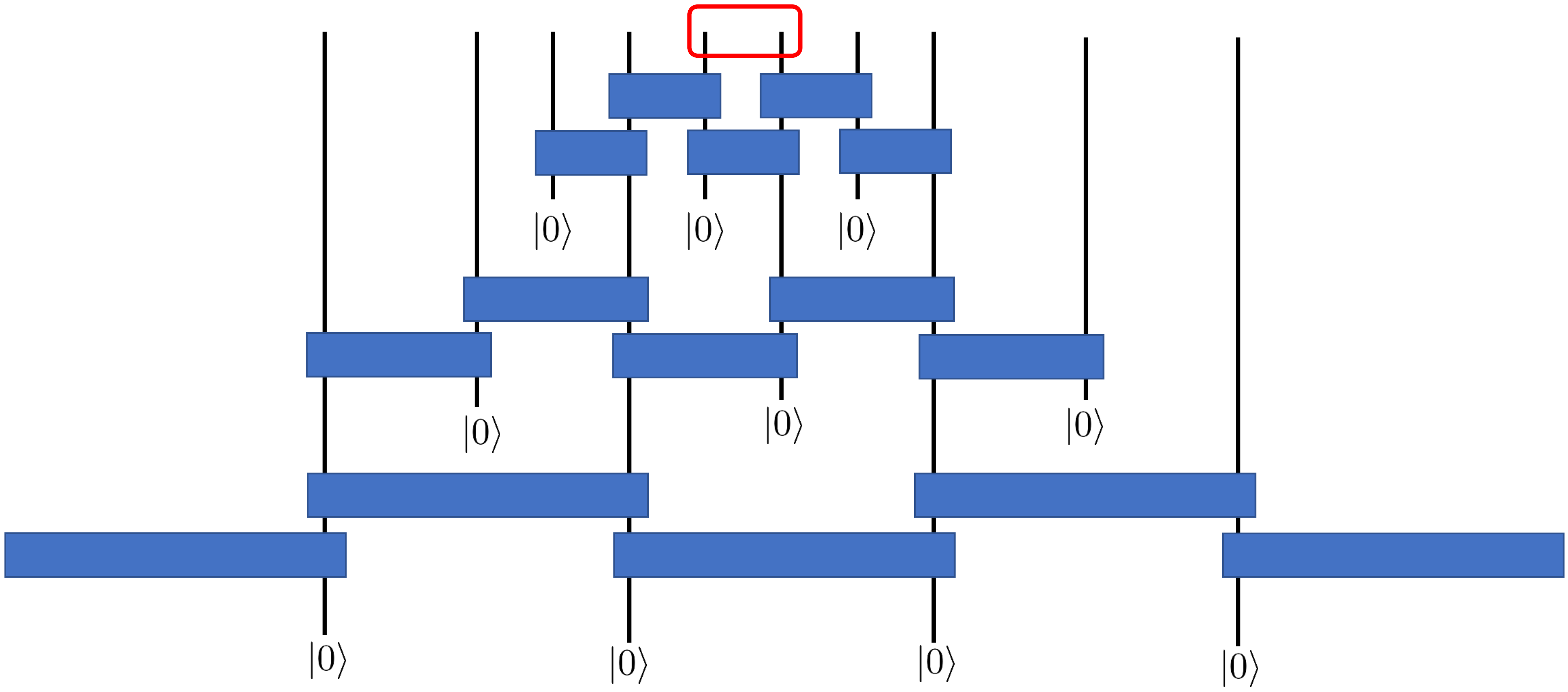

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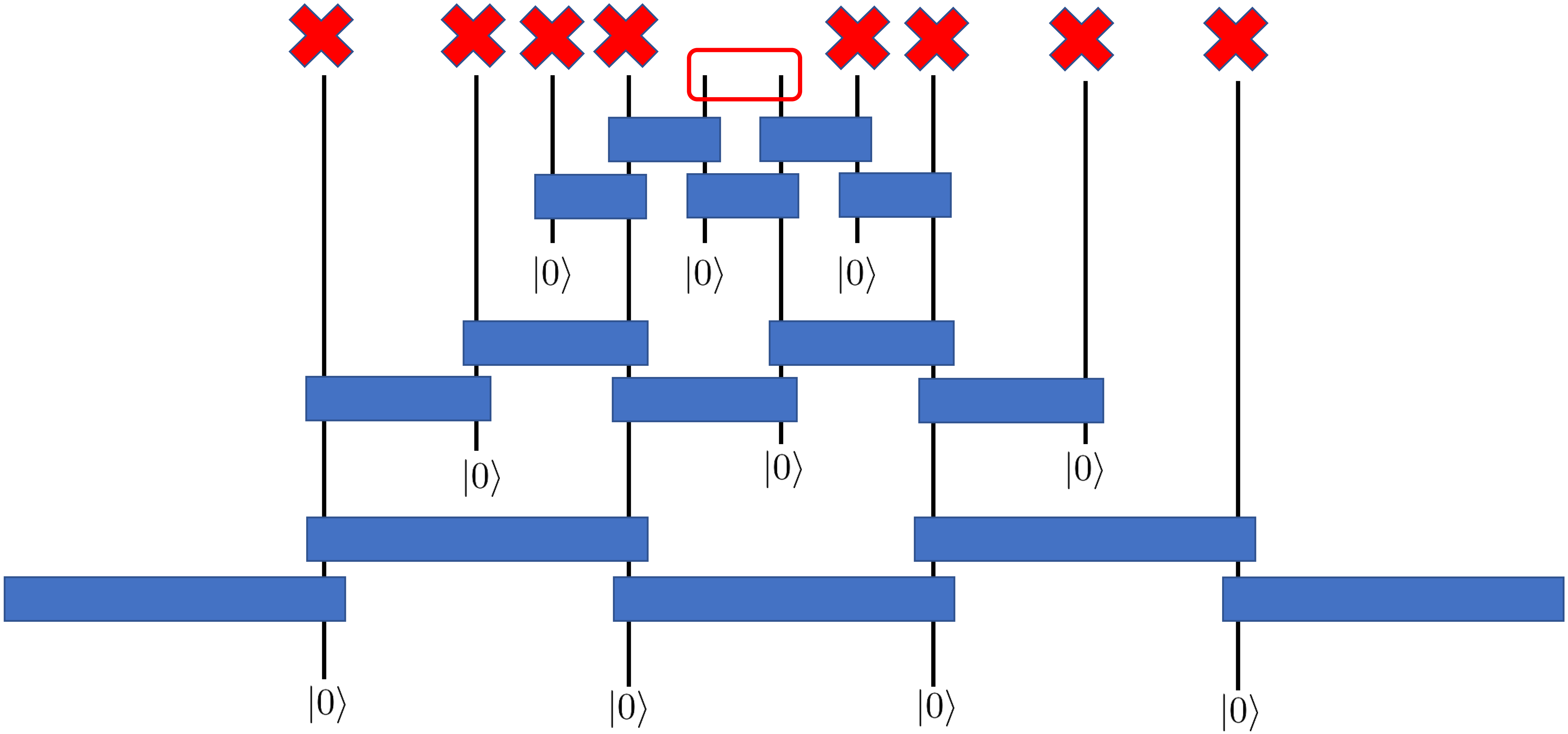

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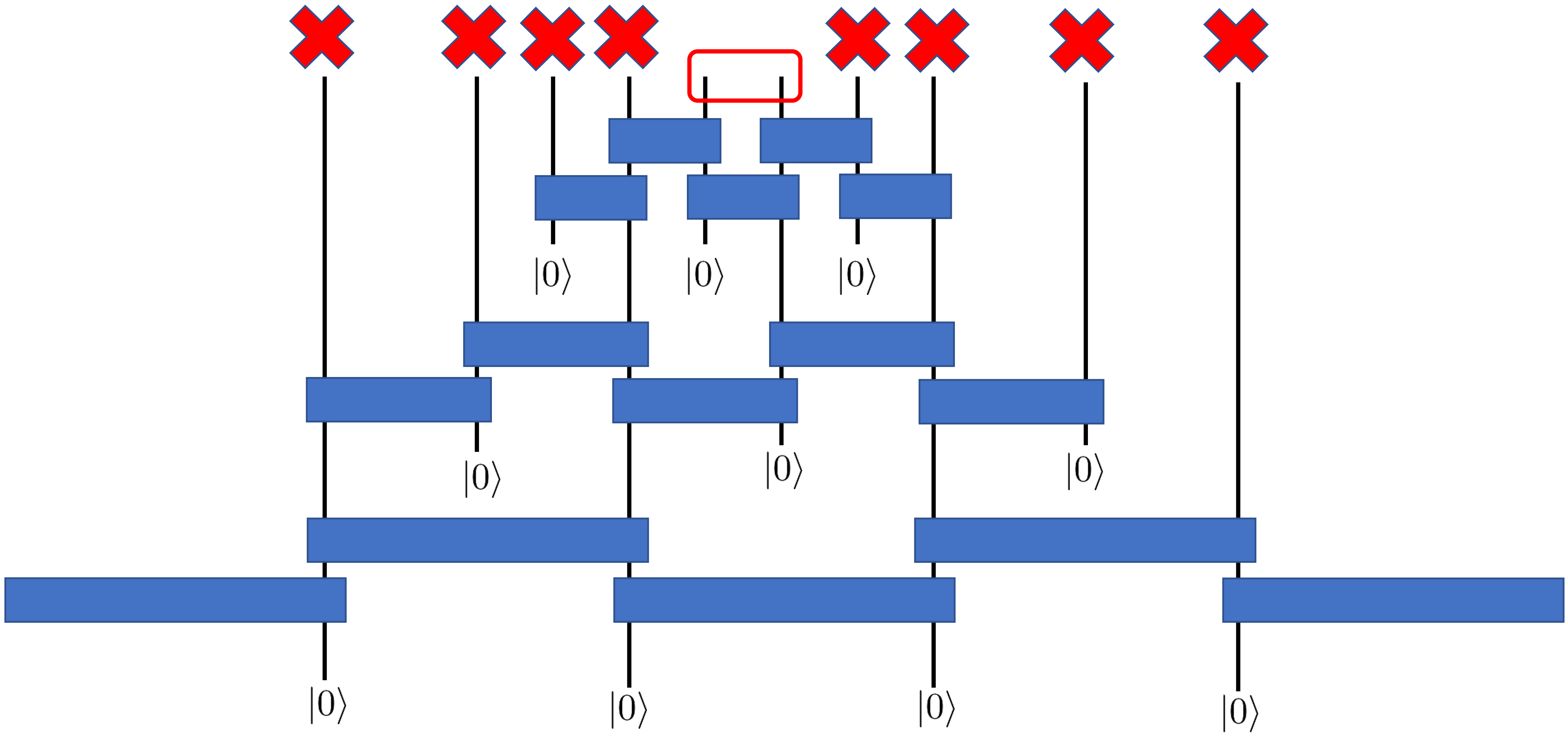


Final state is given by many (< log L) iterations of open system dynamics on O(1) supersites

Bad news and good news

- Bad: So far it seems that (outside of one dimension) the needed bond dimensions are just out of reach classically, e.g. 2D MERA cost χ^{16}; e.g. one needs to diagonalize a very big matrix, but the matrix is so big it doesn't even fit into memory (and isn't obviously sparse); [Evenbly-Vidal] however, I'm not ruling out classical improvements

- Good: If the tensor network calculation can be instantiated physically, in a quantum system, then the calculation might actually be feasible

[Kim-S in progress]

$$\chi \to \log \chi$$

Comments (1)

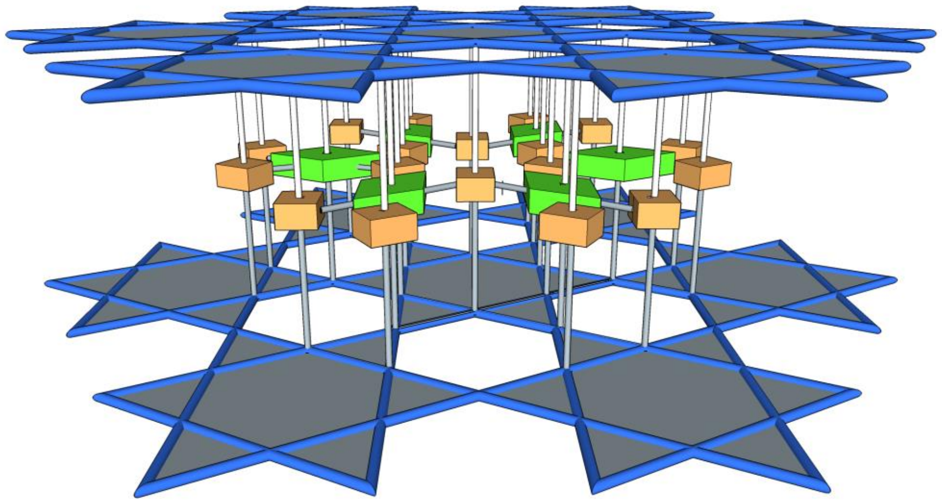
- State of O(1) supersites (even spatially separated supersites) can be obtained by acting on O(1) supersites with O(log L) quantum channels (channel = tensor in extra supersites in known states, act with unitaries, discard extra supersites)
- Memory: No scaling with system size! Time: Only log scaling with system size (and may have fixed point)!
- Key questions:
  - How big does the bond dimension χ need to be?
  - How complex are the unitaries in the network?

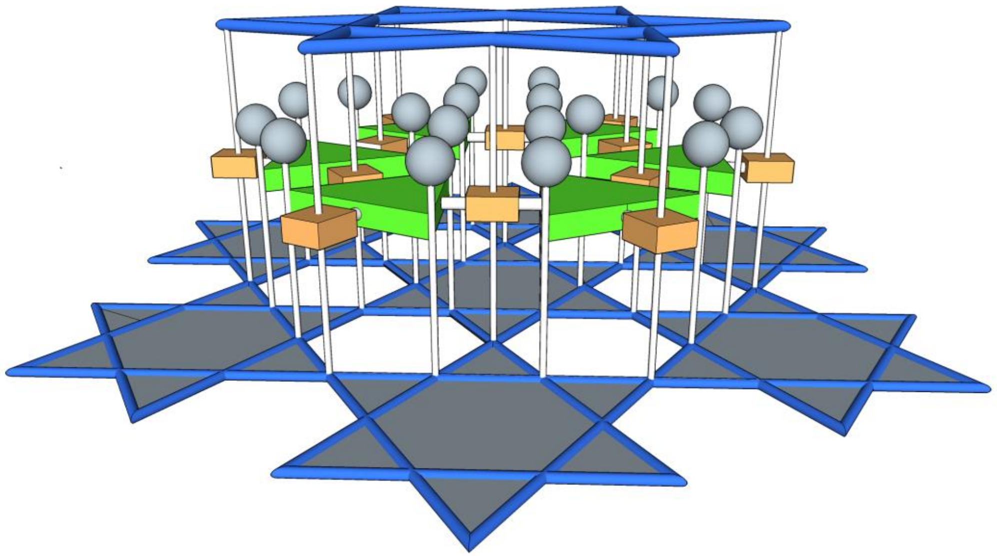
Comments (2)

- Scheme continues to work in higher dimensions with the same scaling with system size; main difference is in the local architecture

$$H = \sum_r \left(-\sigma_r^x + e^{-\alpha \sigma_r^z \sum_{r' \in nn(r)} \sigma_{r'}^z} \right)$$

[S-Xu-McGreevy 1602.06271]





Comments (3)

- Qubits and gates need not be ideal; it seems some noise can be tolerated (but our simulations are preliminary)

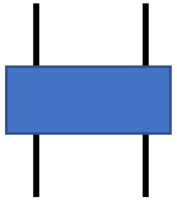
- Physical Argument 1: Expanding universe dilutes errors

[S-McGreevy]

- Physical Argument 2: Long time behavior of open system can be stable to noise given a spectral gap, e.g. as arises in a CFT

[Kim-S in progress, e.g. Cubitt et al.,]

So how complex are the unitaries?



- “Trivial” gapped and lattice scale: adiabatic preparation → finite depth circuit, depth ~ correlation length + poly(|log(error)|)
- “Topological” gapped: adiabatic expansion → finite depth circuit for each layer, depth ~ correlation length + poly(|log(error)|)
- Gapless free fermions: wavelet construction → depth ~ |log(error)|
- Holographic states: use recent complexity/geometry conjectures → depth ~ central charge (probably + |log(error)|) [Susskind, Brown et al. 1512.04993]

- Conjecture: This behavior is generic for a wide class of systems

What should we actually aim to do?

- Interesting target Hamiltonians: non-integrable spin chain (1+1 D), quantum Ising model (2+1 D), frustrated spin model (2+1 D)?, …
- Variationally minimize the energy, compute (measure) local physical data → translation invariance? if not, stronger system size dependence for variational optimization; usual variational problems, e.g. local minima, may remain
- Alternatively, implement known circuit (obtained somehow) which cannot be addressed classically, compute (measure) local physical data

THANKS!

$$N_{\mathrm{qubits},\mathcal{N}=4} \stackrel{?}{=} 10 \times (16^3) \times (2+6+8) \times N^2$$