Visualization of Qutrit States

Vinod K. Mishra, Ph.D.

US Army Research Laboratory, Aberdeen, MD 21005

vinod.k.mishra.civ@mail.mil

Abstract

The qutrit comes next in complexity after qubit as a resource for quantum information processing. The qubit density matrix can be easily visualized using Bloch sphere representation of its states. This simplicity is unavailable for the 8-dimensional state space of a qutrit. A qutrit density matrix is of order 3 and depends on 8 parameters. There are many attempts at visualizing the state space of a qutrit [1-4].

(i) Using all 8 Gell-Mann matrices, which form a complete set for expressing 3 X 3 SU(3) matrices

(ii) Using 6 Gell-Mann matrices supplemented by different matrices in place of two diagonal ones

(iii) Bloch matrices and their principal minors

The Bloch matrix approach leads to many complicated constraints on the qutrit parameters. Previous work in this area has focused on understanding the structure of 8-dimensional parameter spaces through 2- and 3-sections. On the other hand the 3-dimensional Bloch ball state space of qubit are easily visualizable..We present a scheme for representing qutrit state space in 3-dimensions.

The most general qutrit density matrix can be represented using SU (3) invariant form as $\rho = \frac{1}{3} I_3 + \vec{\boldsymbol{n}}.\vec{\boldsymbol{\lambda}}$**.** Here I_3 is 3×3 unit matrix, , $\vec{\mathbf{n}} = (n_1, n_2, \ldots, n_8)$ are 8 real parameters, and , $\vec{\boldsymbol{\lambda}} = (\lambda_1, \lambda_2, \ldots, \lambda_8)$ are 3×3 Gell-Mann matrices. An alternative representation of the qutrit density matrix is based on the symmetric part of the 2 qubit Bloch matrix representing a spin-1 state. The parameters in this representation are connected to spin-1 observables: $\omega_i = < S_i^2 > = Tr(\rho S_i^2)$, $a_i = < S_i > = Tr(\rho S_i)$, and $q_k = < S_i S_j + S_j S_i > = Tr\{\rho(S_i S_j + S_j S_i)\}, k \neq i, j$. Comparison of this parametrization of qutrit density matrix with the earlier one based on the Gell-Mann matrices gives new constraints: (i) $\omega_1 \geq 0, \omega_2 \geq 0, \omega_3 \geq 0$, (ii) $Tr(\rho) = 1$ or $\omega_1 + \omega_2 + \omega_3 = 1$, (iii) $\vec{\boldsymbol{n}}.\vec{\boldsymbol{n}} \leq \frac{1}{3}$ or $\sum_{i=1}^{3} \left\{\omega_i^2 + \frac{1}{2}(a_i^2 + q_i^2)\right\} \leq 1$, and (iv) $\det \rho \geq 0$ or $\frac{1}{4}\sum_{i=1}^{3} \omega_i (a_i^2 + q_i^2) \leq 3\omega_1\omega_2\omega_3$. Earlier attempts at visualizing the general constraints given above have focused on 8-dimensional Cartesian space.

The Main Result: We combine these constraints to define 3-dimensional vectors such that $\vec{\boldsymbol{u}}.\vec{\boldsymbol{u}} \leq 1$, $\vec{\boldsymbol{v}}.\vec{\boldsymbol{v}} \leq 1$, and further combine them to arrive at a vector $K = |\vec{\boldsymbol{K}}| = \frac{\mathbf{1}}{\mathbf{2cos}\frac{\alpha}{2}}\sqrt{u^2 + v^2 + 2uvcos\alpha}$,. The states are confined inside the positive octant of a 3 dimensional sphere of unit radius as the vector components are always positive by construction. The pure qutrit states are characterized by points on the surface whose distance from the center is given by $K = 1$. The mixed states reside inside the volume interior to the “pure qutrit state surface”. This is similar to the Bloch ball of a qubit. Details will be given in the poster.

REFERENCES: [1] S. K. Goyal, B. N. Simon, R. Singh, and S. Simon, arXiv: 1111.4427, [2] Pawel Kurzynski, Adrian Kolodziejski, Wieslaw Laskowski, and Marein Markiewicz,, arXiv:1601.07361v1