Measurement uncertainty relations for finite observables

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Given a set of non-commuting sharp observables what is the best joint measurement approximating them? A quantitative answer to such a question is given by a measurement uncertainty relation (see Figure 1). As such questions naturally arise in many applications within the fields of quantum cryptography and quantum computing, we want to provide a poster wrapping up the main results of our current works $[1, 2, 3]$ which are tackling the above question.

The basic method used for finding an optimal joint measurement is semidefinite programming, which we apply to arbitrary finite collections of projective observables on a finite dimensional Hilbert space. The quantification of errors is based on the Wasserstein distance build on an (arbitrary) cost function, which assigns a penalty to getting result x rather than y, for any pair (x, y) of measurement outcomes. There are different ways to form an overall figure of merit from the comparison of distributions. We consider three, which are related to different physical scenarios. The first figure of merit compares the distances between the marginals of a joint measurement and a reference observable for every input state. The second is a figure of merit that just tests on the states for which a "true value" is known in the sense that the reference observable yields a definite outcome. Thirdly, we can also measure an error as a single expectation value by comparing the two observables on the two parts of a maximally entangled state. All three error quantities have the property that they vanish if and only if the tested observable is equal to the reference. We provide a general method to efficiently compute the POVM representation of an optimal joint measurement and uncertainty relations with arbitrary numerical precision. Moreover we give analytical lower bounds based on the norm of certain commutators.
In cryptographic settings one often takes the discrete metric as a cost function and hence obtain the total variational distance as an error measure. Here we compute the exact error trade-off for different pairs of observables that are typically arising as decoding maps in data locking protocols.

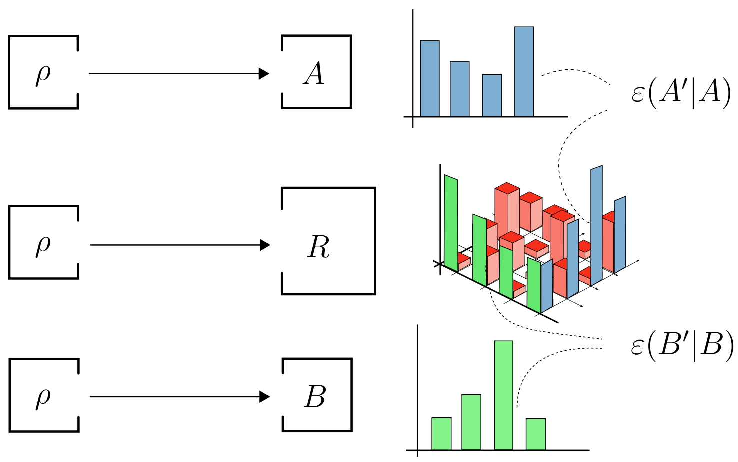

Figure 1. Basic setup of measurement uncertainty relations. The approximate joint measurement R is shown in the middle, with its array of output probabilities. The marginals A' and B' of this array are compared with the output probabilities of the reference observables A and B, shown at the top and at the bottom. The uncertainties $\varepsilon(A'|A)$ and $\varepsilon(B'|B)$ are quantitative measures for the difference between these distributions.

References:

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