Shortcuts to Quantum Network Routing

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Once a quantum network is built, we will need to route quantum messages effectively to their destination. Communication in the classical case is based on a fixed network. With quantum communications we can improve on the network by distributing quantum entanglement, whereas noise and coherence times challenge the effectiveness of conventional routing. In our work we take the first step in studying protocols for quantum communications on a very high level. Concretely, we will consider a simplified model in which each quantum network node can store a small number of qubits, create EPR pairs with their immediate neighbours, and perform entanglement swapping operations to create long-distance entanglement [BVK98]. We will take the perspective that creating pairwise entanglement is itself a time consuming process. For example, if heralding is used to create entanglement between solid state qubits [Ber+13], or if entanglement distillation needs to be performed first [Ben+96]. However, we will take the –for now– highly optimistic view that the lifetime of the qubits at each network node is long. Indeed, in this first step, we will not put a limit on the lifetime of the qubits at all.

Given that establishing entanglement takes time, and that each network node can store only few qubits, we consider the question of how to distribute EPR pairs between network nodes ahead of time to deal with subsequent communication requests. A naive approach would be to distribute an EPR pair between every pair of nodes, resulting in a storage size of $O(N^2)$ qubits at each node in a network of N nodes. When a communication request then arrives, we immediately have a link given by the shared EPR pair to teleport the desired qubit. The main contribution of our work are protocols that reduce the space complexity from $O(N^2)$, which is too large for the currently small quantum memories, to a more manageable $O(\log N)$. Our protocols achieve this with at most $O(\log N)$ entanglement swapping operations per request. These operations may be noisy themselves, thus we wish to minimize the number of swapping operations [Dür+99]. Classically, we find a shortest path to minimize the number of swaps.

We consider two very simple network topologies, nodes on a ring and on a sphere. These are two natural topologies when considering a city ring of network nodes or a network of quantum satellites distributed around the earth. We define a recursive process to create a graph matching the network. By this process we gain insight in a possible distribution of EPR pairs over the network. Distance is measured in the number of entanglement swaps necessary to communicate. Classically, the maximum distance between nodes (the diameter) in both the ring and the sphere is $O(N)$. By distributing entanglement we reduce this to $O(\log N)$. Though more than the naive approach, our procedure only requires a quantum memory of size $O(\log N)$ per node and thus is more feasible to implement. Additionally, we give distributed routing algorithms for finding the shortest path by exploiting the network structure. These algorithms have an exponentially lower time and space complexity than their general variants.

The allocation of entangled links is similar to the allocation of circuits in circuit switched routing. Yet, the fact that entangled links are consumed upon use and can in principle be established between any pair of nodes lends a dynamic character to the problem not present in classical networks. It is even possible to anticipate on future requests and prepare resources in advance to reduce the number of entanglement swaps required for network requests. We capture the time necessary to distribute entanglement with a high level analysis in time units per node. Naively distributing entanglement takes at least $O(N)$ time steps. Our protocols require at most $O(\log N)$ time steps of parallel operations to distribute entanglement throughout the network. Likewise, this affects how quickly the network can restore entanglement that was consumed upon use or has become unusable over time.

Our analysis does remain at a very high level and must still be brought closer to real world devices. To gain a first insight into routing in quantum networks we have not yet taken into account many of the physical properties of quantum links. For example, we assume that qubits have long lifetimes, that entanglement is perfect, and that there are no errors in the operations. Furthermore, we have not considered the possibility of multi-partite states and the undoubtedly interesting interplay between protocols for routing and for creating entangled links. Since EPR pairs can only be used once before they have to be re-established it will also be necessary to investigate how robust the network is when handling multiple simultaneous requests.

[†] The time complexity of the classical routing algorithm is

	Classical	Naive Q.	Ring	Sphere
Diameter	$O(N)$	$O(1)$	$O(\log N)$	$O(\log N)$
Quantum Memory	—	$O(N^2)$	$O(\log N)$	$O(\log N)$
Entangl. Distribution	—	$O(N)$	$O(\log N)$	$O(\log N)$
Routing (time/node)	$O(N \log N)^\dagger$	$O(1)$	$O(1)$	$O(\log^7 N)$
Routing (space/node)	$O(N^2)$	$O(1)$	$O(1)$	$O(\log^2 N)$

Table 1: A comparison of a classical protocol (using Dijkstra's algorithm [KT06]), a naive quantum communication protocol and our contributions for the ring and sphere networks. The structure of the network in our contribution allows for decreasing the diameter and keeping the quantum memory size feasible. At the same time we can route using decentralised routing algorithms and easily redistribute entanglement after use.

Nevertheless, we show that routing with few qubits of storage in the nodes is in principle possible in networks with a number of nodes exponential in the storage size. The results of our main contributions are summarised in Table 1. We also believe the methods used for distributing EPR pairs of the network can be generalised to other types of networks such as a grid network. If a network graph can be generated by a similar recursive procedure we believe it is possible to apply our sphere routing procedure and achieve the same performance.

References

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