# Nonlocal Correlations of Entangled Two-Qudit States using Energy-Time Entangled Photons

Sacha Schwarz,[1,*] Bänz Bessire,[1] Alberto Montina,[2] Stefan Wolf,[2] Yeong-Cherng Liang,[3] and André Stefanov[1]

[1] *Institute of Applied Physics, University of Bern, 3012 Bern, Switzerland*
[2] *Facoltà di Informatica, Università della Svizzera italiana, 6900 Lugano, Switzerland*
[3] *Department of Physics, National Cheng Kung University, Tainan 701, Taiwan*

We present an experimental scheme for creating, characterizing, and manipulating freuqency-entangled qudits by shaping the energy spectrum of entangled photons. We perform various quantum information protocols with dimension up to four and show how the method can be extended to distribute entangled state of high dimension, open the way to quantum key distribution with large qudits.

The physical principle of nonlocality, i.e. the fact that entangled quantum systems can exhibit correlations between measurement outcomes that are not Bell-local [1, 2], is in many different quantum information processing applications of fundamental interested. A prominent example of this is the possibility to perform quantum key distributions whose security is guaranteed without relying on any assumption about the measurements being performed nor the quantum state prepared [3]. Similarly, nonlocality is also essential ingredient for the self-testing [4] of quantum apparatus directly from measurement statistics. More recently, the paradigm of *device-independent quantum information* [2, 5] — where the analysis of quantum information is based solely on the observed correlations — has also been applied in the context of randomness expansion [6], randomness extraction [7], dimension-witnessing [8], as well as robust certification [9], classification [10] and quantification [11] of entanglement.

For all these tasks, an imperative step is to certify that the observed correlation is not Bell-local — a task that is often achieved through the violation of *Bell inequalities* [1]. Achieving a solid understanding of the quantum violation of Bell inequalities is thus an important step towards the development of novel device-independent quantum information processing tasks, for instance. To date, however, the bulk of such studies have focused on the simplest Clauser-Horne-Shimony-Holt [12] (CHSH) Bell scenario, namely, one involving only two parties, each performing two binary-outcome measurements. While more complicated Bell scenarios, such as those involving more parties [13, 14], or more measurement settings [15–17] or more measurement outcomes [18, 19] have also been individually considered, scenarios involving a combination of these have so far received relatively little attention (see, however, Refs. [20, 21]).

Stimulated through this motivation, we experimentally study nonlocal correlations via the generation and manipulation of entangled photonic qudits. Due to their low decoherence rate, photons provide a robust carrier of entanglement, usually produced by the nonlinear interaction of spontaneous parametric down-conversion (SPDC) [22]. The coherence of this process, together with conservation rules, can generate entanglement in the finite Hilbert space of polarization modes and in continuous spaces for transverse modes or for frequency modes. Regarding quantum key distribution, the latter is the most suitable degree of freedom in order to distribute entanglement over a large distance through optical fibers.

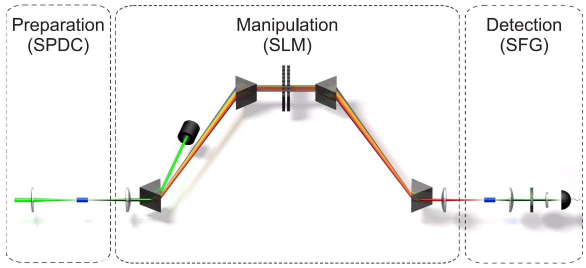


FIG. 1. Schematic overview of the experimental setup. We generate entangled photon pairs via spontaneous parametric down-conversion (SPDC) by pumping a nonlinear crystal with a quasi-monochromatic laser. The generated biphoton state, entangled in the energy-time degree of freedom, is subsequently manipulated via a prism-based pulse shaper including a spatial light modulator (SLM) as reconfigurable modulation device. Finally, coincidences are detected through sum-frequency generation (SFG) via a second nonlinear crystal.

As depicted in Fig. 1, we make use of an experimental setup derived from a classical pulse-shaping arrangement, containing a spatial light modulator (SLM) as reconfigurable modulation tool. It allows for full coherent control over entangled qudits through coherent modulation of the broadband biphoton spectrum, generated via SPDC. Up to first order in perturbation theory, the corresponding biphoton state is described by

$$|\psi\rangle = \int_{-\infty}^{\infty} d\Omega\, \Lambda(\Omega)\, \hat{a}_i^\dagger(\Omega)\, \hat{a}_s^\dagger(-\Omega)\, |0\rangle_i\, |0\rangle_s, \qquad (1)$$

where the leading order vacuum term is omitted. By acting on the composite vacuum state $|0\rangle_i\,|0\rangle_s$, the operators $\hat{a}_{i,s}^\dagger(\Omega)$ create the idler (i) and signal (s) photon at relative frequency $\Omega$ (with respect to $\frac{\omega_{p,c}}{2}$). Since we assume a continuous pump field as well as degenerate center frequencies $\omega_{c,i} = \omega_{c,s} = \frac{\omega_{p,c}}{2}$, for idler and signal photon, respectively, the joint spectral amplitude (JSA), in general being a two-dimensional function denoted by $\Lambda(\Omega_i, \Omega_s)$, simplifies to $\Lambda(\Omega)$ with $\Omega \equiv \Omega_s = -\Omega_i$, as used in Eq. 1. The detection of the photon pair is realized by optical ultrafast coincidence of the two entangled photons in sum-frequency generation (SFG). Intrinsically, the setup is phase stable and can implement any

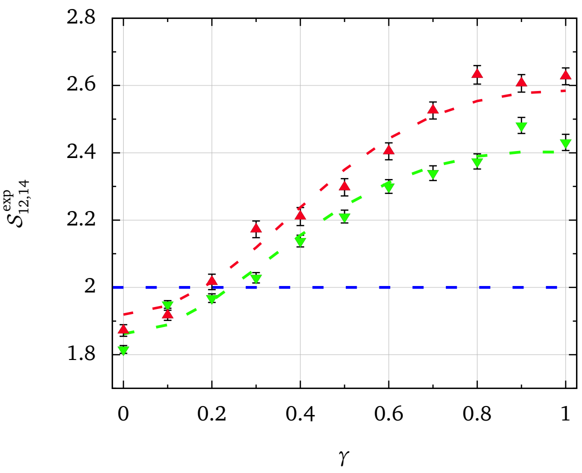


FIG. 2. Experimentally measured Bell parameters $\mathcal{S}^{\text{exp}}_{12,14}$ based on a one-parameter family of entangled two-qutrit states. $\mathcal{I}^{\text{max}}_{14}$ is a ternary-outcome Bell inequality that is maximally violated by a maximally entangled two-qubit state, whereas $\mathcal{I}^{\text{max}}_{12}$ is maximally violated by performing a genuine POVM, or in other words, *non-projective measurements* on a partially entangled two-qubit state.

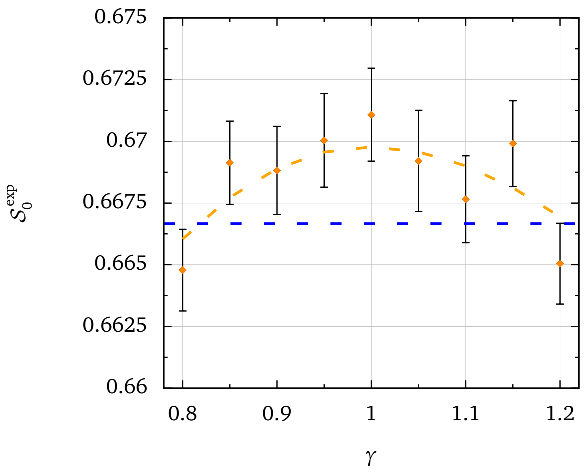


FIG. 3. Experimentally measured Bell parameters $\mathcal{S}^{\text{exp}}_{0}$ based on a one-parameter family of entangled two-qutrit states, $I_3^+$ is the only known Bell inequality with maximal quantum violation achieved by a maximally entangled two-qutrit state.

discretization of the continuous spectrum. Due to high flexibility by means of the SLM, we are able to encode with the same setup energy-bin and time-bins, but also other energy modes, like Schmidt modes of entangled states [23].

Several quantum information protocols are realized in the energy-bin basis, including quantum state tomography up to $d = 4$ [24], quantum state estimation [25] and violation of Bell inequalities. By means of the present setup, we studied the violation of the CHSH inequality employing entangled qubits and the Collins-Gisin-Massar-Linden-Popescu (CGLMP) inequality with entangled qutrits [26]. Further we considered a more complex Bell scenario, incorporating three ternary-outcome measurements, reported in [27]. Surprisingly, all these inequalities involving only genuine ternary-outcome measurements can be violated maximally by some two-qubit entangled states, such as the maximally entangled two-qubit state. This gives further evidence that in analyzing the quantum violation of Bell inequalities, or in the application of the latter to device-independent quantum information processing tasks, the commonly-held wisdom of equating the local Hilbert space dimension of the optimal state with the number of measurement outcomes is not necessarily justifiable. In Fig. 2 and Fig. 3, we show exemplarily experimentally measured Bell parameters for the two inequalities $\mathcal{I}^{\text{max}}_{12}$ and $\mathcal{I}^{\text{max}}_{14}$ by means of a one-parameter family of entangled two-qubit states and the inequality $I_3^+$ by means of a one-parameter family of entangled two-qutrit states, respectively. The corresponding states are described by

$$|\psi(\gamma,\gamma')\rangle = \frac{|0\rangle_{\text{A}}|0\rangle_{\text{B}} + \gamma|1\rangle_{\text{A}}|1\rangle_{\text{B}} + \gamma'|2\rangle_{\text{A}}|2\rangle_{\text{B}}}{\sqrt{1+\gamma^2+\gamma'^2}}, \qquad (2)$$

with $\gamma' = 0$ for qubits and $\gamma' = 1$ for qutrits in the range of $\gamma \in [0,1]$.

Complementary to the experimental examination of Bell inequality violations in different scenarios, we quantified in [28] nonlocality based on the sets of experimentally determined correlations. On one side by its distance to the local polytope and on the other side by the minimal amount of classical communication required to simulate the correlations, we call the corresponding measure *nonlocal capacity*. Referring to the latter, we went a step further and take up in [29] the question of how we can find the optimal experimental setup for a given quantum state, i.e. the setup that maximizes the nonlocality of correlations with respect to communication complexity. Besides the evaluation of the nonlocal capacity formulated as a convex optimization problem, we point out in [29] that the use of the strength of Bell inequality violations as measure of nonlocality comes with some drawbacks like the infeasibility to determine the whole set of Bell inequalities already for a few measurements or the ambiguity stemming from the choice of the normalization of the Bell coefficients. Employing communication complexity for quantifying nonlocality instead allows foremost the determination of the optimal experimental setup for a given state in any bipartite Bell scenario, i.e. for any number of measurements and/or outputs. Moreover, the optimization problem may also be used for solving an open question concerning Werner states.

In conclusion, we present an experimental scheme for studying nonlocal correlations of two-qudit states, treating different quantum information protocols. In addition, we elaborate suitable quantifiers for entanglement and nonlocality, directly applied on experimentally determined correlations. As an outlook, we aim to achieve in ongoing work a better optical resolution, a higher signal-to-noise ratio setup and an experimental arrangement in which the idler and signal photon can be manipulated individually. The finite optical resolution at the SLM plane leads to the aforementioned, unwanted overlap between spectral components and therefore forbids us from having well-separated subsystems. This overlap can be reduced by incorporating a prism based pulse shaper in which the single lens imaging system is replaced by a 4f-imaging arrangement, the latter providing a significantly better imaging resolution quality. Further, since SFG in a single nonlinear crystal is a local detection process with both photons taking the same optical path through the experimental setup, another goal in future is to have a detection scheme based on two delocalized nonlinear crystals. The up-conversion process between the idler and signal photon is then triggered by a strong seed pulse in optical gating such that the up-converted photons are finally detected in coincidence electronically.

* sacha.schwarz@iap.unibe.ch

[1] J. Bell, Physics **1**, 195 (1964).
[2] N. Brunner, D. Cavalcanti, S. Pironio,V. Scarani, and S. Wehner, Rev. Mod. Phys. **86**, 419 (2014).
[3] J. Barrett, L. Hardy, A. Kent, Phys. Rev. Lett. **95**, 010503 (2005); A. Acín, N. Brunner, N. Gisin, S. Massar, S. Pironio, and V. Scarani, Phys. Rev. Lett. **98**, 230501 (2007).
[4] X. Wu, Y. Cai, T. H. Yang, H. N. Le, J.-D. Bancal, and V. Scarani, Phys. Rev. A **90**, 042339 (2014); K. F. Pál, T. Vértesi, and M. Navascués, *ibid.*, 042340 (2014).
[5] V. Scarani, Acta Phys. Slovaca **62**, 347 (2012).
[6] R. Colbeck, PhD Dissertation, University of Cambridge (2006), arXiv:0911.3814 (2009); R. Colbeck and A. Kent, Journal of Physics A: Mathematical and Theoretical **44**, 095305 (2011).
[7] K.-M. Chung, Y. Shi, and X. Wu, arXiv:1402.4797 (2014).
[8] N. Brunner, S. Pironio, A. Acín, N. Gisin, A. A. Méthot, and V. Scarani, Phys. Rev. Lett. **100**, 210503 (2008).
[9] J.-D. Bancal, N. Gisin, Y.-C. Liang, and S. Pironio, Phys. Rev. Lett. **106**, 250404 (2011).
[10] N. Brunner, J. Sharam, and T. Vértesi, Phys. Rev. Lett. **108**, 110501 (2012).
[11] T. Moroder, J.-D. Bancal, Y.-C. Liang, M. Hofmann, and O. Gühne, Phys. Rev. Lett. **111**, 030501 (2013).
[12] J. F. Clauser, M. A. Horne, A. Shimony, and R. Holt, Phys. Rev. Lett. **23**, 880 (1969).
[13] Y.-C. Liang, D. Rosset, J.-D. Bancal, G. Pütz, T. J. Barnea, and N. Gisin, Phys. Rev. Lett. **114**, 190401 (2015).
[14] R. F. Werner and M. M. Wolf, Phys. Rev. A **64**, 032112 (2001); M. Żukowski and Č. Brukner, Phys. Rev. Lett. **88**, 210401 (2002).
[15] P. M. Pearle, Phys. Rev. D **2**, 1418 (1970); S. L. Braunstein and C. M. Caves, Ann. Phys. **202**, 22 (1990).
[16] N. Brunner and N. Gisin, Phys. Lett. A **372**, 3162 (2008).
[17] K. F. Pál and T. Vértesi, Phys. Rev. A **79**, 022120 (2009).
[18] D. Collins, N. Gisin, N. Linden, S. Massar, and S. Popescu, Phys. Rev. Lett. **88**, 040404 (2002).
[19] D. Kaszlikowski, L. C. Kwek, J.-L. Chen, M. Żukowski, and C. H. Oh, Phys. Rev. A **65**, 032118 (2002).
[20] S. Massar, S. Pironio, J. Roland, and B. Gisin, Phys. Rev. A **66**, 052112 (2002).
[21] B. Grandjean, Y.-C. Liang, J.-D. Bancal, N. Brunner, and N. Gisin, Phys. Rev. A **85**, 052113 (2012).
[22] T. Wihler, B. Bessire, and A. Stefanov, J. Phys. A: Math.Theo. **47**, 245201, 2014.
[23] B. Bessire, C. Bernhard, T. Feurer, and A. Stefanov, New J. Phys. **16**, 033017, 2014.
[24] C. Bernhard, B. Bessire, T. Feurer, and A. Stefanov, Phys. Rev. A **88**, 032322, 2013.
[25] S. Lerch and A. Stefanov, Opt. Lett. **39**, 5399, 2014.
[26] S. Schwarz, B. Bessire, and A. Stefanov, Int. J. Quant. Inform. **12**, 1560026, 2014.
[27] S. Schwarz, B.Bessire, A. Stefanov, and Y.-C. Liang, New J. Phys. **18**, 035001, 2016.
[28] C. Bernhard, B. Bessire, A. Montina, M. Pfaffhauser, A. Stefanov, and S. Wolf, J. Phys. A: Math. Theo. **47**, 424013, 2014.
[29] S. Schwarz, A. Stefanov, S. Wolf, and A. Montina, arXiv:1602.05448.