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Optimal and Secure Measurement Protocols for Quantum Sensor Networks

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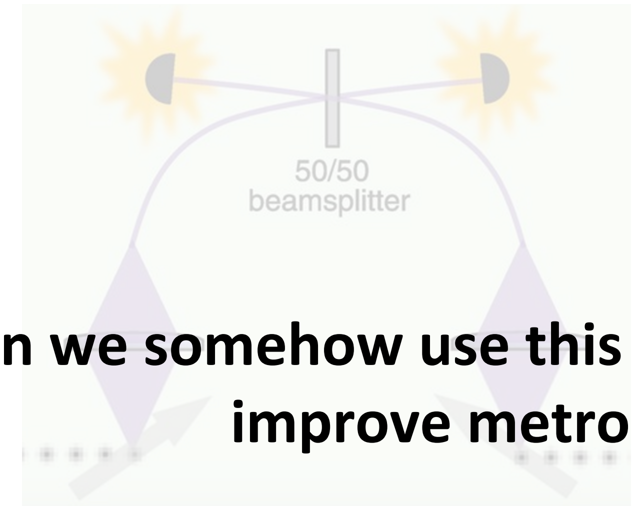
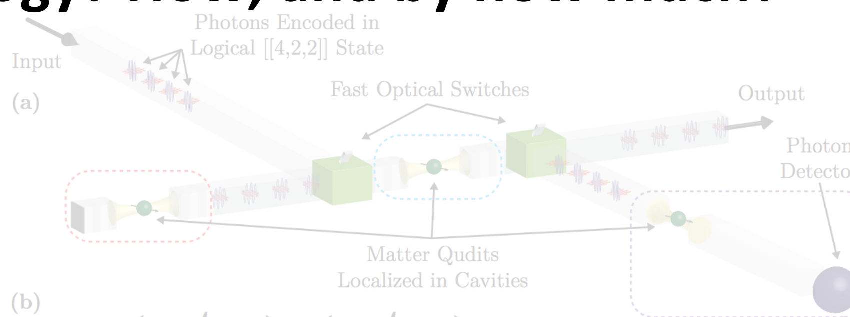
Motivation: Distributed Entanglement



Monroe, Kim 2013

Can we somehow use this distributed entanglement across space to improve metrology? How, and by how much?



Glaudell 2015, Taylor group

Motivation: Quantum Measurement

- Suppose we want to measure a quantity θ, using N systems

$$H = \frac{1}{2}\theta \sum_{i=1}^{N} \sigma_i^z$$

- "Standard quantum limit": $\Delta\theta \propto \frac{1}{\sqrt{N}}$
  - No entanglement in probe
- "Heisenberg limit": $\Delta\theta \propto \frac{1}{N}$
  - Best scaling consistent with uncertainty principle

Bollinger 1996

Inhomogeneous Measurement

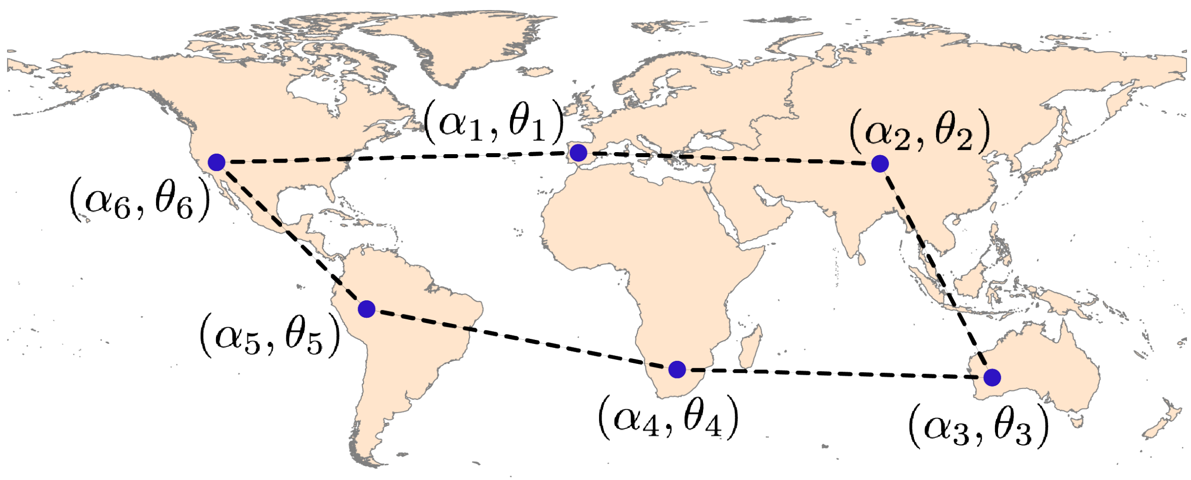
- Assume qubits are subject to Hamiltonian

$$H = \frac{1}{2} \sum_{i=1}^{N} \theta_i \sigma_i^z$$

- Now what can we measure?
- Interesting case: linear combination

$$Q = \sum_{i=1}^{N} \alpha_i \theta_i = \vec{\alpha} \cdot \vec{\theta}$$

Networks for Measurement



- Every point has an associated weight and an associated parameter

$$(\alpha_i, \theta_i)$$

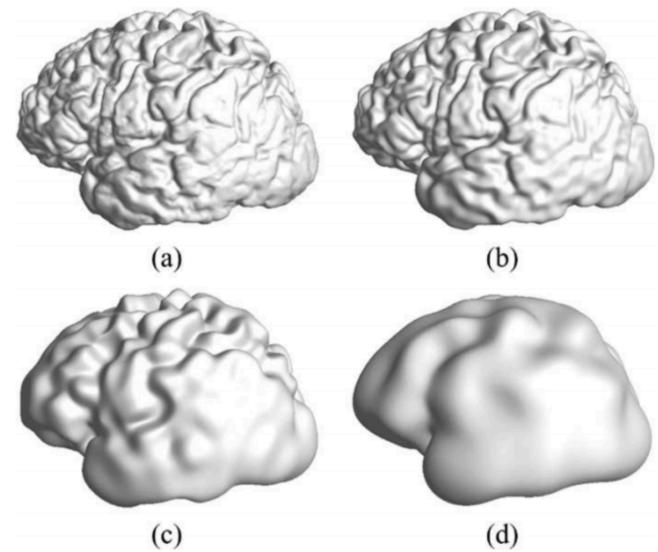
Weight (we pick) → α_i Field (unknown) → θ_i

- Network is capable of both quantum and classical communication

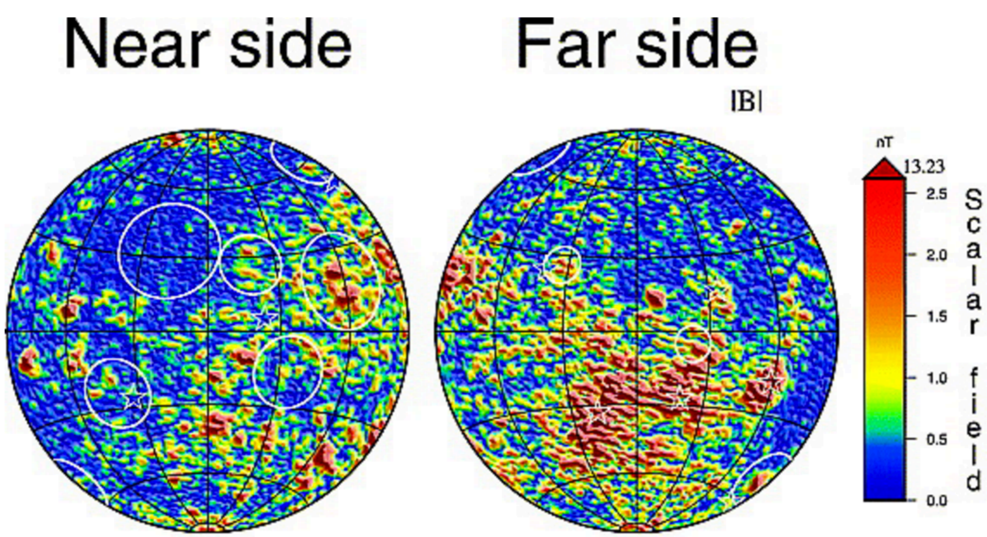
$$Q = \sum_{i=1}^{N} \alpha_i \theta_i = \vec{\alpha} \cdot \vec{\theta}$$

Why A Linear Combination?

- Isolate single mode of the field
- Geology, magnetometry, geodesy, neuroscience…



Gu 2004



Purucker 2010

Standard Quantum Limit (Networks)

- Suppose no entanglement between sites
- Variance = weighted sum of individual variances
- Give everyone best possible measurement:

$$\Delta\theta_i = 1/t$$

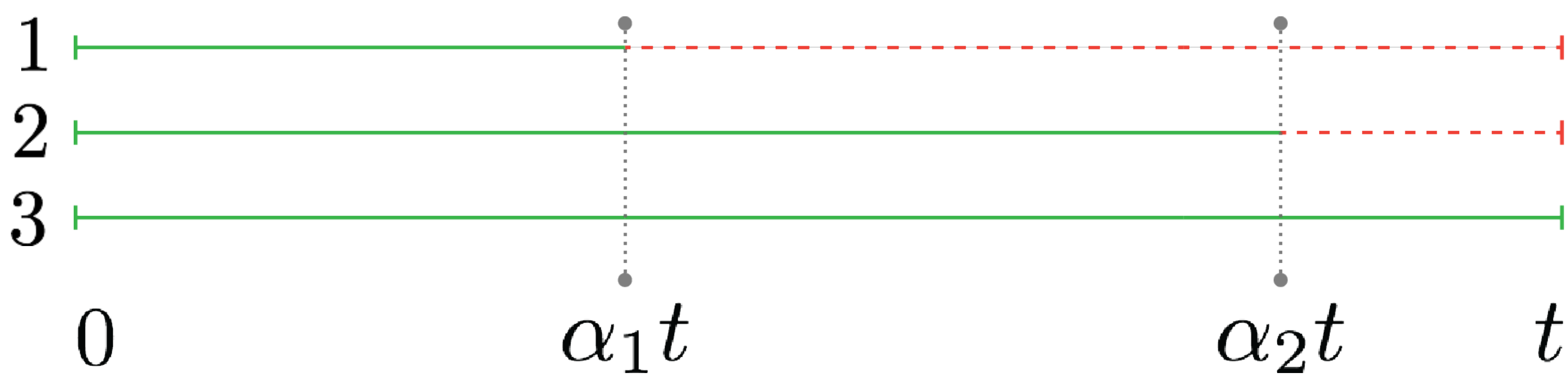
$$\Delta Q$$

Entangled Protocol

- Every station shares a GHZ state

$$|0\rangle^{\otimes N} + |1\rangle^{\otimes N}$$

- Set largest $\alpha_i \to 1$ WLOG
- Evolve qubits for time proportional to weight



Entangled Protocol: Making the Measurement

- Final state is

$$|\psi(t)\rangle = |000\ldots\rangle + e^{-i\overbrace{\sum \alpha_j \theta_j}^{Q} t}|111\ldots\rangle$$

- Measuring in $|\pm\rangle$ at every site extracts Q

(nonlocal observable is secure against subverted qubits)

$$\left\langle \prod_{i=1}^{N} \sigma_i^x \right\rangle \sim \cos Qt$$

$$\implies \Delta Q \geq \frac{1}{t}$$

Optimal (in Fisher information), even given arbitrary external control

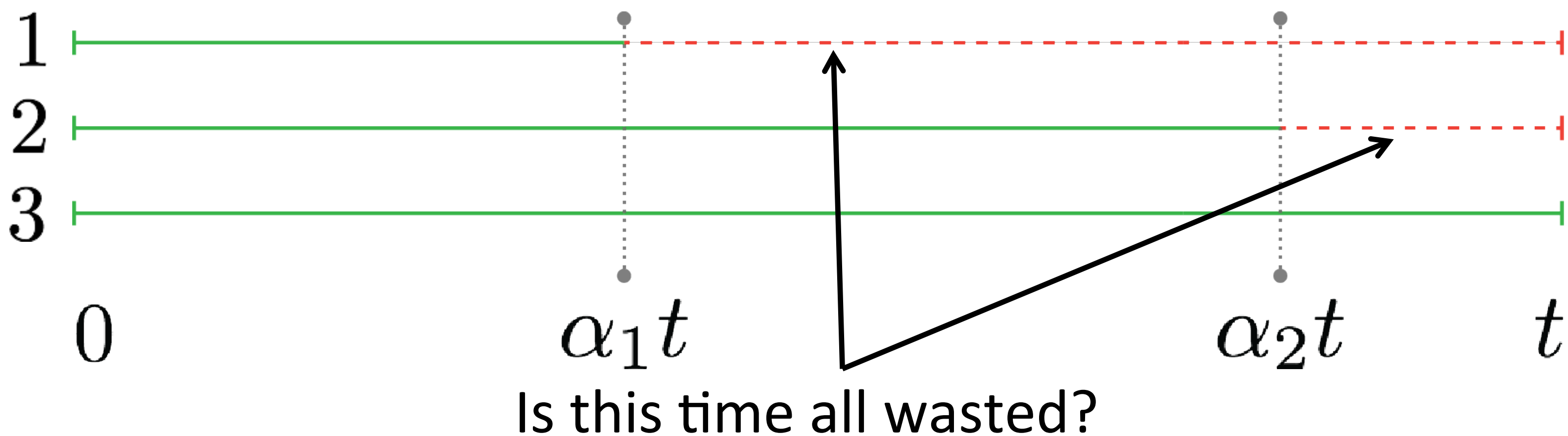
Bollinger 1996, Boixo 2007

Entangled Protocol: Performance

	Entangled	Disentangled
Bounds	$\Delta Q \geq \frac{1}{t}$	$\Delta Q \geq \frac{\|\vec{\alpha}\|}{t}$
In components	1	$\sqrt{\alpha_1^2 + \alpha_2^2 + \cdots}$
Single site	$1/t$	$1/t$
Average	$1/Nt$	$1/\sqrt{N}t$

Entangled Protocol: Shortcomings

- Lots of qubits don't do much in our protocol – they are "lazy"



So how can this be **optimal?**

Why Simple = Optimal?

- Consider alternative protocol on two qubits

$$|00\rangle + |11\rangle$$

Evolve for $\alpha_1 t$

$$|00\rangle + e^{-i\alpha_1(\theta_1+\theta_2)t}|11\rangle$$

Disentangle

$$(|0\rangle + |1\rangle)\left(|0\rangle + e^{-i\alpha_1(\theta_1+\theta_2)t}|1\rangle\right)$$

Evolve for $\alpha_2 t$

$$\left(|0\rangle + e^{-i(\alpha_2-\alpha_1)\theta_1 t}|1\rangle\right)\left(|0\rangle + e^{-i(\overbrace{\alpha_1\theta_1+\alpha_2\theta_2}^{Q})t}|1\rangle\right)$$

- Measures both sum and individual θ_1?

Why Simple = Optimal?

- Problem is no *a priori* way to leverage this
- Let's play a game
  - I'm thinking of the sum of two numbers
  - You need to guess the sum

A + 13 = ?

- Real situations have prior info, however
  - Asymptotically in repetitions of the protocol, can just measure until we match that

What Next?

- Non-asymptotic region – Bayesian schemes
  - Incorporate prior info
- Noise
- Nonlinear functions
- Develop applications further
- Experiments – anyone who can do entanglement and apply diagonal Hamiltonian

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